Magnetized plasma plumes: physics, open problems

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Plumes generated by magnetic nozzles

- A magnetic nozzle (MN) is the convergent-divergent magnetic topology created by an axisymmetric set of coils or permanent magnets,
- A MN is supposed to:
  - channel and accelerate a plasma plume → reduce plasma backflow
  - produce (magnetic) thrust in a way similar to a solid deLaval nozzle does with a hot, neutral gas.
- Advantages:
  - MN has no walls → no heat loads, no erosion
    - MN shape is adaptable → thruster throttling (Isp vs thrust)
      → 3D MN → thrust vector control
- The MN is being proposed as the main acceleration stage of several new electromagnetic thrusters featuring quasi-axial magnetic topologies.
Plasma thrusters with MN (I)

- **The Helicon Plasma Thruster (HPT)**
  - High plasma density \(10^{18} - 10^{20}\text{ m}^{-3}\)
  - Prototypes in the 50 W – 50 kW range, but still low efficiency: \(\eta_T \ll 0.5\)

- **AF-MPD**
  - DC discharge
  - High power (10 kW – 200 kW); \(\eta_T < 0.5\)

- **VASIMR**
  - A HPT enhanced with an ion-cyclotron-resonance heater stage
  - Requires much higher magnetic field
  - High power (100 kW – few MW); \(\eta_T < 0.72\)
Plasma thrusters with MN (II)

- **The Electron Cyclotron Resonance Thruster**
  - Similar to HPT but different plasma-wave interaction

- All these thrusters share **no external neutralizer**

- All, except AFMPDT, have no electrodes

- **HEMPT and DCFT** (variants of Hall thrusters) include a MN but this should have a minor role, since the plasma is already expanded

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**HEMPT (THALES)**

**ECRA (ONERA)**

**HEMPT (THALES)**

**DCFT (MIT)**
MN chanelling effect

- HPT05 prototype (EP2-SENER) with the MN-coil turned

OFF

ON
When the MN principle was proven?

- The magnetic/solid nozzle analogy was demonstrated experimentally in 1969.
- A few prototypes with MNs were studied in the 1970s-1980s.
- The emergence of research on new thrusters in the last decade has boosted the interest in MN physics and performances.
- Also, MN coupled to plasma sources are being used for plasma manufacturing processes and supersonic wind tunnels.

Andersen et al. 1969
What do we need to understand of MN physics?

Study is focused on **MN divergent region**, outside the thruster chamber

- Convergent region is inside chamber usually, and magnetic confinement there is strongly coupled with chamber processes (ionization, heating, wall-interaction,...)

- **2D supersonic expansion** features
  - Can a quasi-1D model describe MN physics? (Very limited way)

- Effectiveness of **magnetic confinement**
  - Must ions be magnetized?

- Plasma **energy conversion** process from ‘internal’ to ’axial kinetic’ one

- Influence of the **type of internal energy** in the above process (electron/ion, isotropic/ anisotropic, ...) different plasma sources → different MNs

- Downwards **detachment** of plasma from MN

- **Thrust transmission** mechanism:
  - different from solid nozzle (based on pressure on oblique walls)!

- Others, less obvious:  
  - collisionless plume cooling,
  - induced B-field,...
### Main assumptions for core MN model

- Plasma conditions at MN throat known; $R = $ MN radius at throat
- Quasineutral, current-free plasma
  \[ \lambda_{d0}/R \to 0 \quad n_e = n_i \equiv n \]
- Collisionless, fully-ionized plasma $\Omega_{e0}/\nu_{e0} \to \infty$
  - Relevant for propulsive application
- Two-fluid model
- Electron inertia is neglected $m_e/m_i \to 0$
- Fully-magnetized electrons, partially-magnetized ions
  \[ \ell_{e0}/R \to 0 \quad \ell_{i0}/R = O(1) \]
- Effects of induced magnetic field on plasma flow are negligible
  - Moderate plasma density
    \[ \beta_0 \to 0, \text{ with } \beta = \mu_0 n T_e / B^2 \]
- Internal energy is stored isotropically in electrons; $T_i \ll T_e$
  - Application limited to HPT and partially to ECRT
DIMAGNO: a 2D fluid formulation

- Fluid equations
  \[
  \begin{align*}
  \nabla \cdot (n u_i) &= 0, \\
  \nabla \cdot (n u_e) &= 0, \\
  m_i n (u_i \cdot \nabla) u_i &= -en \nabla \phi + en u_i \times B, \\
  0 &= -\nabla p_e + en \nabla \phi - en u_e \times B, \\
  \frac{T_e}{T_0} &= \left( \frac{n}{n_0} \right)^{\gamma - 1}.
  \end{align*}
  \]

- 9 (scalar) equations for 9 variables

- 2 systems of reference
  \[\{1_z, 1_r, 1_\theta\}, \{1_{\parallel}, 1_{\perp}, 1_\theta\}\]

- Streamtubes for
  - B-field
  - Ion fluid
  - Electron fluid
DIMAGNO: a 2D fluid formulation

- Magnetic streamfunction for solenoidal longitudinal B field

\[ \nabla \psi = r B_\perp : \frac{\partial \psi}{\partial z} = -r B_r, \quad \frac{\partial \psi}{\partial r} = r B_z, \]

- B-streamtubes (constant magnetic flux) \( \psi(r, z) = \text{const} \)

- Plasma jet edge corresponds to \( \psi_V(z, r) = \psi(0, R) \)

- Ion/electron streamfunctions

\[ \frac{\partial \psi_j}{\partial z} = -r n u_{rj}, \quad \frac{\partial \psi_j}{\partial r} = r n u_{zj}, \quad j = i, e \]

- i.e. streamtubes: \( \psi_i, \psi_e = \text{const} \):
  
  particle flux of ions/electrons = const

- Electric current density: \( j = e n (u_i - u_e) \)

- It is convenient to split vector variables \( (j, B, u_i, u_e, \ldots) \) in

  - Azimuthal: \( j_\theta = j \cdot 1_\theta \)
  - Longitudinal: \( \tilde{j} = j - j_\theta 1_\theta \)
MN conditions at the throat

- Conditions at MN throat are initial integration conditions
- They are representative of plasma source characteristics
- Here we apply conditions derived from models of a cylindrical HPT source (with strong wall magnetic shielding)

\[
\begin{align*}
n(0, r) & \text{ known, } \quad \phi(0, r) \approx 0, \\
u_{\theta i}(0, r) & \approx 0, \quad u_{\theta e}(0, r) = -\left[ \frac{\partial p_e}{\partial r} \right]_{z=0}, \\
u_{ri}(0, r) & = u_{re}(0, r) \approx 0, \\
u_{zi}(0, r) & = M_0 c_s, \quad (M_0 \geq 1), \\
\int_0^R n u_{ze} r dr & = \int_0^R n u_{zi} r dr
\end{align*}
\]
Conservation/algebraic relations for electrons

- Electron equations are reduced to simple conservation equations

\[
\begin{align*}
    u_{\perp e} & = 0 \\
    n u_{\parallel e} / B & = G_e (\psi) , \\
    h_e(n) - e\phi & = H_e(\psi) . \\
    u_{\theta e} & = - \frac{E_{\perp}}{B} - \frac{1}{enB} \frac{\partial p_e}{\partial \psi} \equiv - \frac{r}{e} \frac{dH_e}{d\psi} ,
\end{align*}
\]

- 1\textsuperscript{st} eq.: electrons remain in the same B-streamsurface
- 2\textsuperscript{nd} eq.: e-streamtubes = B-streamtubes, i.e. $\psi_e(\psi)$
- 3\textsuperscript{rd} eq.: General Boltzmann equilibrium applies in each e-streamsurface
  \[
  \text{[ e.g. for } T_e=\text{const}: \quad n \propto \exp \left( e\phi/T_{e0} \right) \text{ ]}
  \]
- 4\textsuperscript{th} eq.: Azimuthal e-drift is the combination of $E \times B$ and $\nabla p_e \times B$ drifts
  - (Expanding) pressure $\perp$-force = (confining) electric + magnetic $\perp$-forces
  - Electron fluid iso-rotates ($u_{\theta e} / r = \text{const}$) in its helicoidal drift within B-streamsurface

- Functions $G_e$ and $H_e$ are defined at the MN throat
Partial differential equations for ions

- Three hyperbolic differential equations for \( n, u_{zi} \) and \( u_{ri} \)

\[
\begin{align*}
    u_{ri} \frac{\partial \ln n}{\partial r} + u_{zi} \frac{\partial \ln n}{\partial z} + \frac{\partial u_{ri}}{\partial r} + \frac{\partial u_{zi}}{\partial z} & = -\frac{u_{ri}}{r}, \\
    u_{ri} \frac{\partial u_{ri}}{\partial r} + u_{zi} \frac{\partial u_{ri}}{\partial z} + c_s^2 \frac{\partial \ln n}{\partial r} & = -(u_{\theta e} - u_{\theta i}) \Omega_i \cos \alpha + \frac{u_{\theta i}^2}{r}, \\
    u_{ri} \frac{\partial u_{zi}}{\partial r} + u_{zi} \frac{\partial u_{zi}}{\partial z} + c_s^2 \frac{\partial \ln n}{\partial z} & = (u_{\theta e} - u_{\theta i}) \Omega_i \sin \alpha,
\end{align*}
\]

- Plus conservation of azimuthal canonical momentum for \( u_{\theta i} \)

\[
m_i (\tilde{u}_i \cdot \nabla) ru_{\theta i} = -re u_{\perp i} B = -e(\tilde{u}_i \cdot \nabla) \psi
\]

\[
\rightarrow \quad rm_i u_{\theta i} + e\psi = D_i (\psi_i) \quad (D_i \text{ defined at the MN throat})
\]

- Mechanisms governing azimuthal velocities are different for inertial ions and confining electrons
Integrating with method of characteristics

- Mach number for longitudinal ion velocity: \( M = \frac{\dot{u}}{c_s} \)
- If the longitudinal ion velocity is supersonic \((M > 1)\), the above PDEs can be integrated forward-marching with method of characteristic lines.
  - At each point, these are 3: the ion-streamline + 2 Mach-lines
The 2D plasma jet expansion

\( \gamma = 1 \) (isothermal)

- For isothermal case:
  \[-e\phi \propto \ln n \rightarrow \infty \text{ at } |\vec{r}| \rightarrow \infty\]
  which is nonphysical

- For non-isothermal:
  \[-e\phi(\infty, 0) = \frac{\gamma}{\gamma - 1} T_0,\]
  \[u_i(\infty, 0) = c_s \sqrt{M_0^2 + \frac{2}{\gamma - 1}},\]
Ion separation

- Electron streamlines are bended radially into B-lines by full-magnetization
- Ion streamlines are bended radially mainly by the electric field

\[ m_i n (u_i \cdot \nabla) u_i = -en\nabla\phi + enu_i \times B, \]

  - Ions are weakly magnetized in most of the MN
- The ambipolar electric field is self-built in order to preserve quasineutrality (but not current ambipolarity, \( j = 0 \)).
  - This E-field is not strong enough to make i-tubes = B-tubes
Plasma detachment

- Ion separation increases dramatically after turning point of plasma jet.
- When ions become hypersonic, the i-tubes become conical.
- Very small amount of plasma momentum turns back toward the thruster.
- Detachment is mildly sensitive to initial ion magnetization.
- In DIMAGNO, electrons still remain perfectly attach to B-lines, but this is not very relevant since they constitute mainly a neutralizing cloud.
Formation of longitudinal loops

- The separation of ion and electron streamtubes produce necessarily longitudinal electric currents, i.e. $\tilde{j} \neq 0$

- How these longitudinal current loops close on themselves, is out of reach of DIMAGNO: coupling with both the source model and beam far-downstream behavior are needed
Plume divergence

- DIMAGNO analysis has shown that plasma/MN detachment issue is reduced to an assessment of plume divergence, i.e. how much of the momentum gained in the MN is lost radially.
- Plume divergence efficiency function at B=const section is defined as

\[ \eta_{\text{plume}}(B) = \frac{\int_{S(B)} dA n u_{zi}^3}{\int_{S(B)} dA n u_{zi}^2 u_{zi}}. \]

- Better behavior is obtained for
  - low ion magnetization
  - low MN divergence rate
Increment of plasma momentum along MN

- Conversion of internal-into-kinetic plasma energy (or thermal-into-dynamic plasma momentum) does not produce necessarily thrust:
  - double layers are an example
- Plasma momentum equation (once the intermediary E-field is compensated):
  \[ \nabla \cdot (m_i n u_i u_i + p_e \vec{I}) = j \times B. \]
- Increase of axial plasma momentum is due to magnetic axial force. At \( B = \text{const} \) sections:
  \[ F(B) = F_0 + \int_{\mathcal{V}(B)} d\mathcal{V}(-j_\theta) B_r, \]
  with \( F_0 \) = axial momentum at the exit of the plasma source (mostly from pressure on rear wall)
Azimuthal currents & Magnetic thrust

- Magnetic thrust mechanism based on 3rd Newton’s law:
  - (Large) B-field from coils $\Rightarrow$ axial force on plasma (travelling, small) $\theta$-current loops
  - (Small) B-field from plasma $\Rightarrow$ axial force on coil (large) $\theta$-currents
- Currents must counterflow for positive plasma acceleration & thrust
- Ion azimuthal currents are small but paramagnetic and detrimental (they produce drag)

\[
F = \int_{r} \frac{dV}{r} \mathbf{j} \times \mathbf{B}
\]

![Diagram of magnetic nozzle and coil currents](image_url)
On ion & electron dynamics

1) MNs are inefficient for a supersonic beam, since thrust/Isp gain is marginal...and can even be negative

2) The dynamics of ions and electrons are very different (even leaving apart their different magnetization level).
   - Ions are accelerated almost freely by the E-field.
     - Ion fluid and individual ions behave similarly
   - Individual and fluid electron responses are different.
     - Individual electrons are confined by the E-field and most of them bounce back-and-forth along \( B \)
     - Electron fluid velocity along \( B \) is just the contribution of the small fraction of electrons escaping downstream with ions.
     - Azimuthal e-motion combines \( E \times B \) (particle) and \( \nabla p_e \times B \) (fluid)
FUMAGNO: Full ion magnetization model (I)

- Zero ion Larmor radius is a degenerate limit of 2D DIMAGNO
  - Both ions and electrons remain tied to B-streamtubes

- Ion differential equations transform into conservation relations along the magnetic tubes:
  \[
  \begin{align*}
  u_{\perp i} &= 0, \\
  u_{\theta i} &\to 0, \\
  e u_{\theta i} B &= -e E_{\perp} + \kappa_B m_i u_{|| i}^2, \\
  m_i u_{|| i}^2 / 2 + e\phi &= H_i(\psi), \\
  u_{|| e} &= u_{|| i},
  \end{align*}
  \]
  - Notice the asymptotic behavior of the azimuthal velocity and force

- FUMAGNO solution is practically analytical in each B-streamline
  - Clear improvement over other published approximate models

- FUMAGNO yields a good approximation of MN near region and of magnetic thrust

- FUMAGNO is not applicable at all in the far region and to study plasma separation/detachment
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FUMAGNO (II)

Magnetized ions

Unmagnetized ions

$e\phi/T_e$, $\hat{\Omega}_{i0} \to \infty$

$e\phi/T_e$, $\hat{\Omega}_{i0} \to 0$

Magnetized plasma plumes

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Beyond DIMAGNO & FUMAGNO

- DIMAGNO assumptions are being relaxed/modified in order to study more physics and understand more phenomena

- Main open problems:
  - Energetic ions
  - Electron collisionality
  - 3D magnetic nozzle
  - Full coupling with plasma source
  - Induced magnetic field (*)
  - Electron demagnetization (*)
  - Collisionless electron cooling (*)
  - Instabilities, anomalous diffusion?

(*) These effects are important to close properly the MN very-far region
VECMAN: 3D magnetic nozzle

- Thrust vector control mechanism with no movable parts (patented)
  - Set of several coils placed in oblique planes.
- 3D MN has been studied with 3D version of FUMAGNO
  - Good for near region
- Pending problem: 3D-DIMAGNO
  Important issues related to:
  electron & ion confinement, different integration scheme
Energetic ions. Several cases

- Source producing hot ions with $T_i \geq O(T_e)$. Included in DIMAGNO.
  - Uncertainties on throat conds. (coming from those on source model)
  - Ions acquire kinetic energy from both: (1) fluid-dynamic conversion of i-thermal energy; (2) electrothermal conversion of e-thermal energy
  - Electric potential fall depends only on $T_e$.
- (AF-MPDT) Ions acquire swirl (kinetic) energy, $u_{\theta i}$.
  - An AF-MPDT source model is needed before implementation in MN
- (VASIMR) Ions are energized perpendicularly to B and coherently.
  - MN ion equations need to be modified. Source model is needed too.
Helicon source + MN = HPT

- Coupling of MN with plasma source not fully solved
  - Fluid transport
  - Wave-plasma coupling
- Energy transfer and $T_e$ field are key aspects
- Numerical algorithms are very sensitive in MN (short wavelengths, oblique B-field)
Induced magnetic field

- Azimuthal plasma currents induce a longitudinal B-field \( \nabla \times \vec{B}^* = \mu_0 j_\theta 1_\theta \).
- We saw that this field is crucial for thrust transmission.
- It affects MN shape and plasma expansion only if \( \beta = \frac{\mu_0 n T_e}{B^2} = O(1) \).
- The induced field opposes the applied one \( \Rightarrow \) total B-field is weaker, \( \Rightarrow \) MN and plume divergences increase.
- Open issues: (1) problem is elliptic, current loops affect upstream region (2) islands of zero B-field (no magnetization) are created.

Effect of \( \beta \) on B-streamtubes

\( \beta_{a0} = 0.01 \) vs \( \beta_{a0} = 0.05 \)
Electron demagnetization

- The transition (electron) magnetized-to-unmagnetized plume is crucial for the downstream plasma beam closure
- This is to us the main open problem in our MN studies
- DIMAGNO operates under the zero e-inertia, zero e-gyroradius limits.
- Nonzero e-gyroradius effects appear as B decreases.
  - These effects (which include e-inertia and pressure anisotropy) are complex to model consistently
  - Some advances have been made extending DIMAGNO
    - Electron demagnetization tends to increase plasma beam divergence
    - Furthermore, a perturbation approach is possibly non-worthy, since e\textsuperscript{-} become demagnetized further downstream.
- Key point: How to reconcile magnetized and unmagnetized e-models
Electron collisionality

- The key parameter is the (finite) Hall parameter \( \chi = \Omega_e/\nu_e \)
- Collisionality (with neutrals and ions, or anomalous) separate electrons from magnetic streamtubes, which diffuse \text{outwards} with \( u_{\perp e} = \frac{\nu_e}{\Omega_e} u_{\theta e} \)
- Collisional effects increase downstream

These plots were obtained with a fluid/PIC MN code under development

Open issues: (1) effect CEX collisions (2) far region solution in order to verify that MN prevents most of ion backflow
Collisionless electron cooling (I)

- Plasma beam is rarefied $\implies$ few collisions $\implies$ no local thermodynamic equilibrium $\implies$ no justification for isentropic/adiabatic behavior
- Isothermal behavior not applicable to expansion in an infinite region
- Kinetic treatment of electron population is required
  (Much less important for ions)
- This open problem is common to magnetized and unmagnetized plumes
  - It could be easier to treat in magnetized plumes
- Experience shows that there is some electron cooling
  - Phenomenological polytropic fittings are used
- Amount of plume cooling is going to determine final ion energy and interaction with SC surfaces $\implies$ critical issue for SC operators
- Advances have been made with a kinetic model of paraxial convergent-divergent MN with a collisionless, fully magnetized plasma
  - Particles conserve their total energy and their magnetic moment
Collisionless electron cooling (II)

- Electrons from upstream source are reflected back when $v_{\text{axial}} = 0$ (line)
- Different regions in the EVDF space
  - Void region (*white*) $\rightarrow$ Main responsible of cooling
  - Region of free electrons (*green*)
  - Region of reflected electrons (*yellow*)
  - Islands of doubly-trapped electrons (*red*) $\rightarrow$ Origin in collisions or MN formation, Very important in solution $\rightarrow$ How to characterize them?
- Beyond the paraxial case, problem is totally unexplored

$E_c > E_b > E_a$
Collisionless electron cooling (III)

- Interesting comparison with potential fall in Debye sheath:
  \[ \frac{e|\phi_\infty|}{T_e} \approx 0.61 \ln \left( \frac{m_i}{m_e} \right) + 0.78. \]

- Main differences for potential fall in MN:
  - It develops in an infinite region not in a very thin region
  - It is fully quasineutral
  - EVDF is undetermined and much more complex
Collisionless electron cooling (IV)

- A time-dependent paraxial kinetic MN model is being developed to characterize the filling of doubly-trapped electron regions
  - Only partial filling is accomplished
- Open issue: Will occasional collisions eventually fill totally these regions?
Electron cooling takes place in unmagnetized plume too
  – Now electron motion is strongly 2D (not circumscribed to a B-line)

Since a radial electron bouncing is confined and fast:
  – The adiabatic invariant $J_r$ (action integral in $r$ orbits) exists playing a similar role to magnetic moment $\mu$ in magnetized plumes

This model is near finalization and characterizes cooling dependence on plume properties (ion Mach number, propellant type, $T_{e0}$)

Open problem: cooling in high-angle directions. Critical for SC interaction
Our paper publications on MN

1. E. Ahedo & M. Merino, "Two-dimensional supersonic plasma acceleration in a magnetic nozzle", Physics of Plasmas 17, 073501(2010) [PDF] [DOI]
2. E. Ahedo & M. Merino, "On plasma detachment in propulsive magnetic nozzles", Physics of Plasmas 18, 053504 (2011) [PDF] [DOI]
6. M. Merino & E. Ahedo, "Plasma detachment in a propulsive magnetic nozzle via ion demagnetization", Plasma Sources Science and Technology 23, 032001 (2014) [PDF] [DOI]
9. M. Merino & E. Ahedo, "Effect of the plasma-induced magnetic field on a magnetic nozzle", Plasma Sources Science and Technology 25, 045012 (2016) [PDF] [DOI]
11. M. Merino & E. Ahedo, "Contactless steering of a plasma jet with a 3D magnetic nozzle", Plasma Sources Science and Technology, 045012(in press)
Thank you!

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