Magnetized plasma plumes: physics, open problems

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Plumes generated by magnetic nozzles

- A magnetic nozzle (MN) is the convergent-divergent magnetic topology created by an axisymmetric set of coils or permanent magnets,
- A MN is supposed to: channel and accelerate a plasma plume

 \rightarrow reduce plasma backflow

- produce (magnetic) thrust

in a way similar to a solid deLaval nozzle does with a hot, neutral gas.

• Advantages: – MN has no walls \rightarrow no heat loads, no erosion

– MN shape is adaptable \rightarrow thruster throttling (Isp vs thrust)

 \rightarrow 3D MN \rightarrow thrust vector control



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Plasma thrusters with MN (I)



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Plasma thrusters with MN (II)

- The Electron Cyclotron Resonance Thruster
 - Similar to HPT but different plasma-wave interaction
- All these thrusters share no external neutralizer
- All, except AFMPDT, have no electrodes

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 HEMPT and DCFT (variants of Hall thrusters) include a MN but this should have a minor role, since the plasma is already expanded



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MN chanelling effect

ON

HPT05 prototype (EP2-SENER) with the MN-coil turned

OFF





When the MN principle was proven?

- The magnetic/solid nozzle analogy was demonstrated experimentally in 1969
- A few prototypes with MNs were studied in the 1970s-1980s.
- The emergence of research on new thrusters in the last decade has boosted the interest in MN physics and performances.
- Also, MN coupled to plasma sources are being used for plasma manufacturing processes and supersonic wind tunnels

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What do we need to understand of MN physics?

Study is focused on MN divergent region, outside the thruster chamber

- Convergent region is inside chamber usually, and magnetic confinement there is strongly coupled with chamber processes (ionization, heating, wall-interaction,...)
- **2D supersonic expansion** features
 - Can a quasi-1D model describe MN physics? (Very limited way)
- Effectiveness of magnetic confinement
 - Must ions be magnetized?
- Plasma energy conversion process from 'internal' to 'axial kinetic' one
- Influence of the type of internal energy in the above process (electron/ion, isotropic/anisotropic, ...) different plasma sources → different MNs
- Downwards detachment of plasma from MN
- Thrust transmission mechanism:
 - different from solid nozzle (based on pressure on oblique walls) !
- Others, less obvious: collisionless plume cooling,
 - induced B-field,...



Main assumptions for core MN model

- Plasma conditions at MN throat known; R = MN radius at throat
- Quasineutral, current-free plasma

 $\lambda_{d0}/R \to 0$, $n_e = n_i \equiv n$

- Collisionless, fully-ionized plasma $\Omega_{e0}/\nu_{e0} \rightarrow \infty$,
 - Relevant for propulsive application
- Two-fluid model
- Electron inertia is neglected $m_e/m_i
 ightarrow 0$
- Fully-magnetized electrons, partially-magnetized ions $\ell_{e0}/R \rightarrow 0$ $\ell_{i0}/R = O(1)$
- Effects of induced magnetic field <u>on plasma flow</u> are negligible
 - Moderate plasma density

 $\beta_0 \rightarrow 0$, with $\beta = \mu_0 n T_e / B^2$

- Internal energy is stored isotropically in electrons; $T_i \ll T_e$
 - Application limited to HPT and partially to ECRT



DIMAGNO: a 2D fluid formulation

Fluid equations $\nabla \cdot (nu_i) = 0,$ $\nabla \cdot (nu_e) = 0,$ $m_i n (u_i \cdot \nabla) u_i = -en \nabla \phi + en u_i \times B,$ $0 = -\nabla p_e + en \nabla \phi - en u_e \times B,$ $\frac{T_e}{T_0} = \left(\frac{n}{n_0}\right)^{\gamma - 1}$



- 9 (scalar) equations for 9 variables
- 2 systems of reference
 - $\{\mathbf{1}_{\mathbf{z}},\mathbf{1}_{\mathbf{r}},\mathbf{1}_{\theta}\} ~ \{\mathbf{1}_{\parallel},\mathbf{1}_{\perp},\mathbf{1}_{\theta}\}$
- Streamtubes for
 - B-field

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- Ion fluid
- electron fluid





DIMAGNO: a 2D fluid formulation

Magnetic streamfunction for solenoidal longitudinal B field

 $\nabla \psi = r B \mathbf{1}_{\perp} : \ \partial \psi / \partial z = -r B_r, \quad \partial \psi / \partial r = r B_z,$

- B- streamtubes (constant magnetic flux) $\psi(r,z) = \mathrm{const}$
- Plasma jet edge corresponds to $\psi_V(z,r) = \psi(0,R)$
- Ion/electron streamfunctions

$$\frac{\partial \psi_j}{\partial z} = -rnu_{rj} \quad \frac{\partial \psi_j}{\partial r} = rnu_{zj}, \quad j = i, e$$

- i,e - streamtubes: $\psi_i, \psi_e = \text{const:}$ particle flux of ions/electrons = const



- Electric current density: $j = en(u_i u_e)$
 - It is convenient to split vector variables (j, B, u_i, u_e , ...) in
 - Azimuthal: $j_{ heta} = oldsymbol{j} \cdot oldsymbol{1}_{ heta}$
 - Longitudinal: $ilde{\jmath} = oldsymbol{j} j_{ heta} oldsymbol{1}_{ heta}$



MN conditions at the throat

- Conditions at MN throat are initial integration conditions
- They are representative of plasma source characteristics
- Here we apply conditions derived from models of a cylindrical HPT source (with strong wall magnetic shielding)

$$n(0,r) \text{ known}, \quad \phi(0,r) \approx 0, \qquad 0.8$$

$$u_{\theta i}(0,r) \approx 0, \qquad u_{\theta e}(0,r) = -\left[\frac{\partial p_e / \partial r}{e n B}\right]_{z=0}, \qquad 0.6$$

$$u_{ri}(0,r) \approx 0, \qquad u_{\theta e}(0,r) \approx 0, \qquad 0.6$$

$$u_{ri}(0,r) = u_{re}(0,r) \approx 0, \qquad 0.7$$

$$u_{zi}(0,r) = M_0 c_{s0}, \quad (M_0 \ge 1), \qquad \int_0^R n u_{ze} r dr = \int_0^R n u_{zi} r dr$$



Conservation/algebraic relations for electrons

Electron equations are reduced to simple conservation equations

$$\begin{split} u_{\perp e} &= 0\\ nu_{\parallel e}/B = G_e\left(\psi\right),\\ h_e(n) - e\phi &= H_e(\psi).\\ u_{\theta e} &= -\frac{E_{\perp}}{B} - \frac{1}{enB}\frac{\partial p_e}{\partial \mathbf{1}_{\perp}} \equiv -\frac{r}{e}\frac{dH_e}{d\psi}, \end{split}$$

$$\begin{split} h_e(n) &= T_{e0} \ln n, \quad \text{for } \gamma_e = 1, \\ h_e(n) &= T_e \frac{\gamma}{\gamma - 1}, \quad \text{for } \gamma > 1. \end{split}$$

- Functions G_e and H_e are defined at the MN throat

- 1st eq. : electrons remain in the same B-streamsurface
- 2^{nd} eq.: e-streamtubes = B-streamtubes, i.e. $\psi_e(\psi)$
- 3^{rd} eq. : General Boltzmann equilibrium applies in each e-streamsurface [e.g. for T_e =const: $n \propto exp (e\phi/T_{e0})$]
- 4^{th} eq. : Azimuthal e-drift is the combination of $E \times B$ and $\nabla p_e \times B$ drifts
 - (Expanding) pressure \perp -force = (confining) electric + magnetic \perp forces
 - Electron fluid iso-rotates ($\frac{u_{\theta e}}{r} = \text{const}$) in its helicoidal drift within B-streamsurface



Partial differential equations for ions

• Three hyperbolic differential equations for n, u_{zi} and u_{ri}

$$\begin{aligned} u_{ri}\frac{\partial \ln n}{\partial r} + u_{zi}\frac{\partial \ln n}{\partial z} + \frac{\partial u_{ri}}{\partial r} + \frac{\partial u_{zi}}{\partial z} &= -\frac{u_{ri}}{r}, \\ u_{ri}\frac{\partial u_{ri}}{\partial r} + u_{zi}\frac{\partial u_{ri}}{\partial z} + c_s^2\frac{\partial \ln n}{\partial r} &= -(u_{\theta e} - u_{\theta i})\Omega_i \cos \alpha + \frac{u_{\theta i}^2}{r}, \\ u_{ri}\frac{\partial u_{zi}}{\partial r} + u_{zi}\frac{\partial u_{zi}}{\partial z} + c_s^2\frac{\partial \ln n}{\partial z} &= (u_{\theta e} - u_{\theta i})\Omega_i \sin \alpha, \end{aligned}$$

• plus conservation of azimuthal canonical momentum for $u_{\theta i}$

$$m_i(\tilde{u}_i \cdot \nabla) r u_{\theta i} = -reu_{\perp i} B = -e(\tilde{u}_i \cdot \nabla) \psi$$

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 $\rightarrow rm_i u_{\theta i} + e\psi = D_i(\psi_i)$ (*D_i* defined at the MN throat)

 Mechanisms governing azimuthal velocities <u>are different</u> for inertial ions and confining electrons



Integrating with method of characteristics

- Mach number for longitudinal ion velocity: $M = \tilde{u}/c_s$
- If the longitudinal ion velocity is supersonic (M > 1), the above PDEs can be integrated forward-marching with method of characteristic lines.
 - At each point, these are 3: the ion-streamline + 2 Mach-lines



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The 2D plasma jet expansion





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- For isothermal case: $-e\phi \propto \ln n \to \infty$ at $|\vec{r}| \to \infty$ which is nonphysical
- For non-isothermal:

$$-e\phi(\infty, 0) = \frac{\gamma}{\gamma - 1}T_0,$$
$$u_i(\infty, 0) = c_{s0}\sqrt{M_0^2 + \frac{2}{\gamma - 1}},$$





lon separation

- Electron streamlines are bended radially into B-lines by full-magnetization
- Ion streamlines are bended radially mainly by the electric field

 $m_i n \left(\boldsymbol{u}_i \cdot \nabla \right) \boldsymbol{u}_i = -en \nabla \phi + en \boldsymbol{u}_i \times \boldsymbol{B},$

- Ions are weakly magnetized in most of the MN
- The ambipolar electric field is self-built in order to preserve quasineutrality (but not current ambipolarity, j = 0).
 - This E-field is not strong enough to make i-tubes = B-tubes



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Plasma detachment

- Ion separation increases dramatically after turning point of plasma jet
- When ions become hypersonic, the i-tubes become conical.
- Very small amount of plasma momentum turns back toward the thruster.
- Detachment is mildly sensitive to initial ion magnetization
- In DIMAGNO, electrons still remain perfectly attach to B-lines, but this is not very relevant since they constitute mainly a neutralizing cloud.



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Formation of longitudinal loops

- The separation of ion and electron streamtubes produce necessarily longitudinal electric currents, i.e. $\tilde{j} \neq 0$
- How these longitudinal current loops close on themselves, is out of reach of DIMAGNO: coupling with both the source model and beam fardownstream behavior are needed



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Plume divergence

- DIMAGNO analysis has shown that plasma/MN detachment issue is reduced to an assessment of plume divergence, i.e. how much of the momentum gained in the MN is lost radially.
- Plume divergence efficiency function at B=const section is defined as

$$\eta_{plume}(B) = \frac{\int_{\mathcal{S}(B)} dAnu_{zi}^3}{\int_{\mathcal{S}(B)} dAnu_i^2 u_{zi}}.$$

- Better behavior is obtained for
 - low ion magnetization
 - low MN divergence rate





Increment of plasma momentum along MN

- Conversion of internal-into-kinetic plasma energy (or thermal-into-dynamic plasma momentum) does not produce necessarily thrust:
 - double layers are an example
- Plasma momentum equation (once the intermediary E-field is compensated):

 $\nabla \cdot (m_i n u_i u_i + p_e \bar{\bar{I}}) = j \times B.$

 Increase of axial plasma momentum is due to magnetic axial force. At B=const sections:

$$F(B) = F_0 + \int_{\mathcal{V}(\mathcal{B})} d\mathcal{V}(-j_\theta) B_r,$$

with F_0 =axial momentum at the exit of the plasma source (mostly from pressure on rear wall)

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Azimuthal currents & Magnetic thrust

- Magnetic trust mechanism based on 3rd Newton's law:
 - (Large) B-field from coils \Rightarrow axial force on plasma (travelling, small) θ current loops
 - (Small) B-field from plasma \Rightarrow axial force on coil (large) θ -currents
- Currents must counterflow for positive plasma acceleration & thrust
- Ion azimuthal currents are small but paramagnetic and detrimental (they produce drag)



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On ion & electron dynamics

- 1) MNs are inefficient for a supersonic beam, since thrust/Isp gain is marginal...and can even be negative
- 2) The dynamics of ions and electrons are very different (even leaving apart their different magnetization level).
 - Ions are accelerated almost freely by the E-field.
 - Ion fluid and individual ions behave similarly
 - Individual and fluid electron responses are different.

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- Individual electrons are confined by the Efield and most of them bounce back-and-forth along *B*
- Electron fluid velocity along **B** is just the contribution of the small fraction of electrons escaping downstream with ions.
- Azimuthal e-motion combines $E \times B$ (particle) and $\nabla p_e \times B$ (fluid)

 $u_i(\infty, 0) = c_{s0} \sqrt{M_0^2 + \frac{2}{\gamma - 1}},$ (b)





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FUMAGNO: Full ion magnetization model (I)

- Zero ion Larmor radius is a degenerate limit of 2D DIMAGNO
 - Both ions and electrons remain tied to B-streamtubes
- Ion differential equations transform
 into conservation relations
 along the magnetic tubes:
 $u_{\perp i} = 0,$ $u_{\theta i} \rightarrow 0,$ $e_{u_0}: B = -e_{u_0}$

$$\begin{split} u_{\perp i} &= 0, \\ u_{\theta i} \to 0, \\ e u_{\theta i} B &= -e E_{\perp} + \kappa_B m_i u_{\parallel i}^2, \\ m_i u_{\parallel i}^2 / 2 + e \phi &= H_i(\psi), \\ u_{\parallel e} &= u_{\parallel i}, \end{split}$$

- Notice the asymptotic behavior of the azimuthal velocity and force
- FUMAGNO solution is practically analytical in each B-streamline
 - Clear improvement over other published approximate models
- FUMAGNO yields a good approximation of MN near region and of magnetic thrust
- FUMAGNO is not applicable at all in the far region and to study plasma separation/detachment



FUMAGNO (II)



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Beyond DIMAGNO & FUMAGNO

- DIMAGNO assumptions are being relaxed/modified in order to study more physics and understand more phenomena
- Main open problems:
 - Energetic ions
 - Electron collisionality
 - 3D magnetic nozzle
 - Full coupling with plasma source
 - Induced magnetic field (*)
 - Electron demagnetization (*)
 - Collisionless electron cooling (*)
 - Instabilities , anomalous diffusion?

(*) These effects are important to close properly the MN very-far region



VECMAN: 3D magnetic nozzle

- Thrust vector control mechanism with no movable parts (patented)
 - Set of several coils placed in oblique planes.
- 3D MN has been studied with 3D version of FUMAGNO
 - Good for near region
- Pending problem: 3D-DIMAGNO
 Important issues related to: electron & ion confinement, different integration scheme

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Simulation	Ampere-turn ratios	F/F_0	ψ (deg)	θ (deg)	θ_B (deg)
0	15:0.33:0.33:0.33	1.44	_	0.00	0.00
A	15:1:0:0	1.44	-180.00	5.66	5.76
В	15:0.5:0.5:0	1.44	-120.00	2.86	2.91
A'	15:5:0:0	1.34	-180.00	11.06	11.24
B'	15:2.5:2.5:0	1.34	-120.00	5.61	5.70
	$\begin{array}{c} \text{Simulation} \\ O \\ A \\ B \\ \hline A' \\ B' \end{array}$	$\begin{array}{c c} {\rm Simulation} & {\rm Ampere-turn\ ratios} \\ \hline O & 15:0.33:0.33:0.33 \\ A & 15:1:0:0 \\ \hline B & 15:0.5:0.5:0 \\ \hline A' & 15:5:0:0 \\ \hline B' & 15:2.5:2.5:0 \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

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Energetic ions. Several cases

- Source producing hot ions with $T_i \ge O(T_e)$. Included in DIMAGNO.
 - Uncertainties on throat conds. (coming from those on source model)
 - Ions acquire kinetic energy from both: (1) fluid-dynamic conversion of i-thermal energy; (2) electrothermal conversion of e-thermal energy
 - Electric potential fall depends only on T_e .
- (AF-MPDT) lons acquire swirl (kinetic) energy, $u_{\theta i}$.
 - An AF-MPDT source model is needed before implementation in MN
- (VASIMR) lons are energized perpendicularly to B and coherently.
 - MN ion equations need to be modified. Source model is needed too.



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Helicon source + MN = HPT

- Coupling of MN with plasma source not fully solved
 - Fluid transport
 - Wave-plasma coupling
- Energy transfer and T_e field are key aspects
- Numerical algorithms are very sensitive in MN (short wavelenths, oblique B-field)





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Induced magnetic field

- Azimuthal plasma currents induce a longitudinal B-field $\nabla \times \tilde{B}^* = \mu_0 j_\theta \mathbf{1}_{\theta}$.
- We saw that this field is crucial for thrust transmission
- It affects MN shape and plasma expansion only if $\beta = \frac{\mu_0 nT_e}{B^2} = O(1)$
- The induced field opposes the applied one ⇒ total B-field is weaker, ⇒
 MN and plume divergences increase
- Open issues: (1) problem is elliptic, current loops affect upstream region
 (2) islands of zero B-field (no-magnetization) are created



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Electron demagnetization

- The transition (electron) magnetized-to-unmagnetized plume is crucial for the downstream plasma beam closure
- This is to us the main open problem in our MN studies
- DIMAGNO operates under the zero e-inertia, zero e-gyroradius limits.
- Nonzero e-gyroradius effects appear as B decreases.
 - These effects (which include e-inertia <u>and</u> pressure anisotropy) are complex to model consistently
 - Some advances have been made extending DIMAGNO
 - Electron demagnetization tends to increase plasma beam divergence
 - Furthermore, a perturbation approach is possibly non worthy, since e⁻ become demagnetized further downstream.
- Key point: How to reconcile magnetized and unmagnetized e-models





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Electron collisionality

- The key parameter is the (finite) Hall parameter $\chi = \Omega_e / \nu_e$
- Collisionality (with neutrals and ions, or anomalous) separate electrons from magnetic streamtubes, which diffuse <u>outwards</u> with $u_{1} = \frac{\nu_e}{\mu_0}u_0$
- Collisional effects increase downstream

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- These plots were obtained with a fluid/PIC MN code under development
- Open issues: (1) effect CEX collisions (2) far region solution in order to verify that MN prevents most of ion backflow



Collisionless electron cooling (I)

- Plasma beam is rarefied \Rightarrow few collisions \Rightarrow no local thermodynamic equilibrium \Rightarrow no justification for <u>isentropic/adiabatic behavior</u>
- Isothermal behavior not applicable to expansion in an infinite region
- Kinetic treatment of electron population is required (Much less important for ions)
- This open problem is common to magnetized and unmagnetized plumes
 - It could be easier to treat in magnetized plumes
- Experience shows that there is some electron cooling
 - Phenomenological polytropic fittings are used
- Amount of plume cooling is going to determine final ion energy and interaction with SC surfaces → critical issue for SC operators
- Advances have been made with a kinetic model of paraxial convergentdivergent MN with a collisionless, fully magnetized plasma
 - Particles conserve their <u>total energy</u> and their <u>magnetic moment</u>



Collisionless electron cooling (II)

- Electrons from upstream source are reflected back when v_{axial}= 0 (line)
- Different regions in the EVDF space
 - Void region (white) \rightarrow Main responsible of cooling
 - Region of free electrons (green)
 - Region of reflected electrons (yellow)
 - Islands of doubly-trapped electrons (*red*) → Origin in collisions or MN formation, Very important in solution → How to characterize them?
- Beyond the paraxial case, problem is totally unexplored



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Collisionless electron cooling (III)



$$\frac{e\phi_{sh}}{T_e} \approx 0.5 \ln \frac{m_i}{m_e} - 0.92$$

- Main differences for potential fall in MN:
 - It develops in an infinite region not in a very thin region
 - It is fully quasineutral

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EVDF is undetermined and much more complex



Collisionless electron cooling (IV)

- A time-dependent paraxial kinetic MN model is being developed to characterize the filling of doubly-trapped electron regions
 - Only partial filling is accomplished

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• Open issue: Will occassional collisions eventually fill totally these regions?



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Collisionless electron cooling (V)

- Electron cooling takes place in unmagnetized plume too
 - Now electron motion is strongly 2D (not circumscribed to a B-line)
- Since a radial electron bouncing is confined and fast:
 - The adiabatic invariant J_r (action integral in r orbits) exists playing a similar role to magnetic moment μ in magnetized plumes
- This model is near finalization and characterizes cooling dependence on plume properties (ion Mach number, propellant type, T_{e0})
- Open problem: cooling in high-angle directions. Critical for SC interaction







Our paper publications on MN

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