Asymptotic-Preserving methods for the efficient resolution of anisotropic equations arising in magnetized plasma physics.

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We present a class of numerical methods design to address efficiently the resolution of elliptic or diffusion equations arising in magnetized plasma simulation. Standard discretizations of these problems give rise to system matrices with a condition number increasing with the anisotropy strength. This difficulty can be explained by the singular nature of these problems. Indeed in the limit of infinite anisotropy, these equations degenerate into a system with an infinite amount of solutions. The principle of Asymptotic-Preserving (AP) methods is to manufacture a set of reformulated equations, equivalent to the original problem, in which this limit is regular [1]. The reformulated system remains well posed irrespective of the anisotropy. The matrices issued from standard discretizations of this system have a condition number bounded with respect to the anisotropy strength [2,3,4]. This permits the construction of numerical methods with a computational cost and a precision roughly independent of the anisotropy.

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