



An overview of instabilities capable of inducing
cross-field transport in low-temperature, partially
magnetized discharges

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E × B Plasmas for Space and Industrial Application

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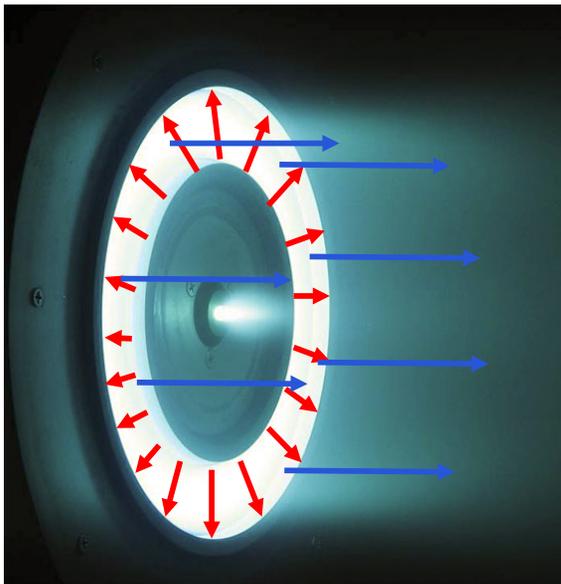
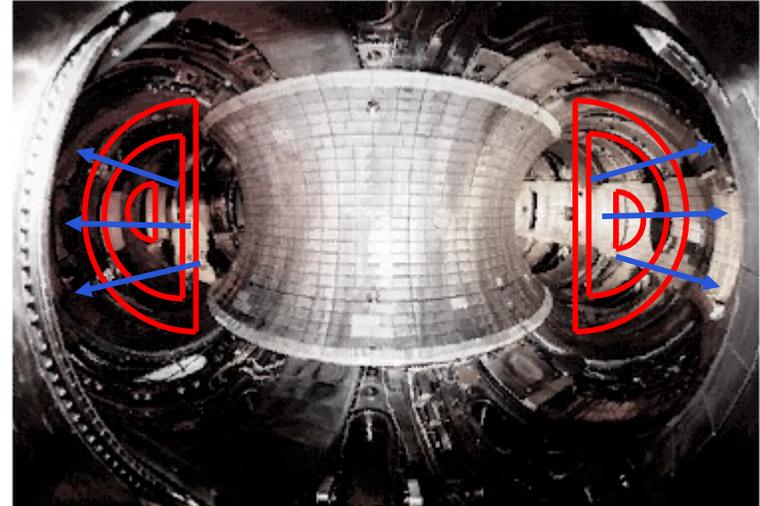
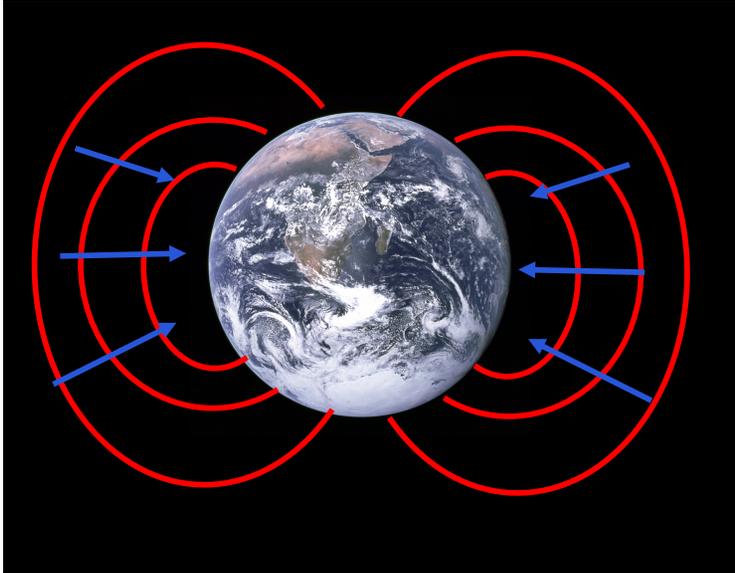


Key Points

- What is the problem of cross field transport in partially magnetized, low-temperature devices?
- How can instabilities contribute to this transport?
- How do we differentiate among the “zoo” of modes that exist in these devices?

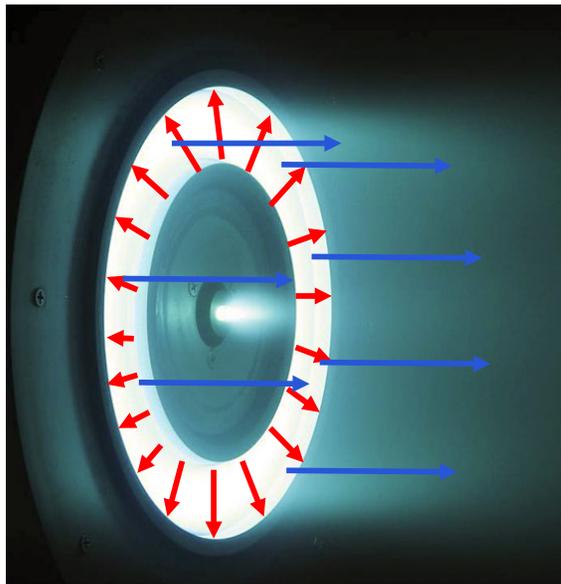
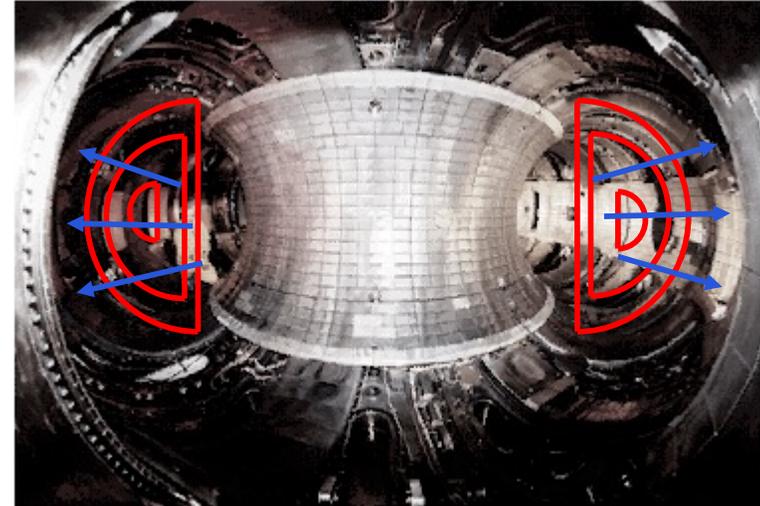
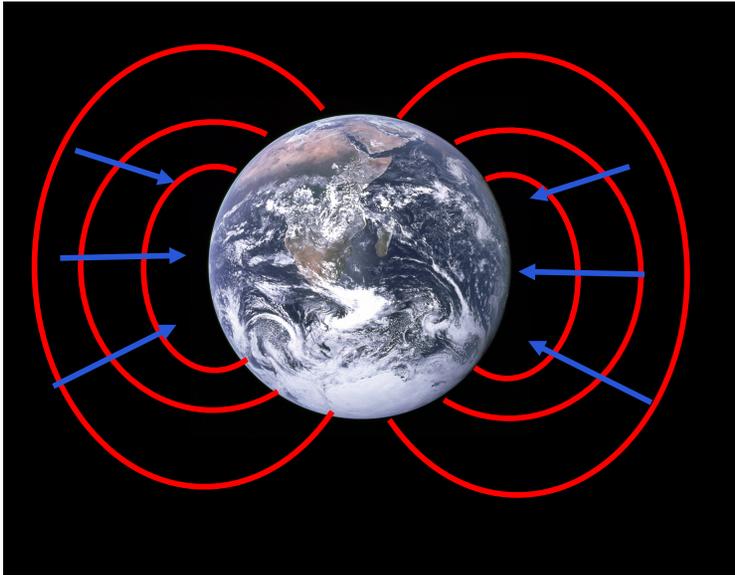


Anomalous Cross-Field Transport Occurs in Many Plasma Systems





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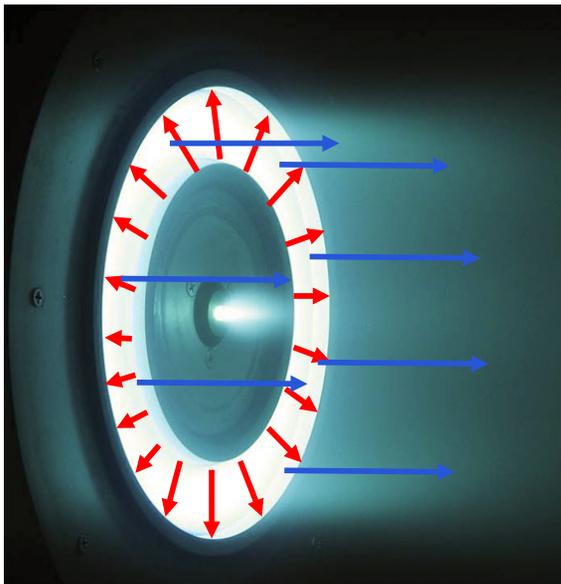
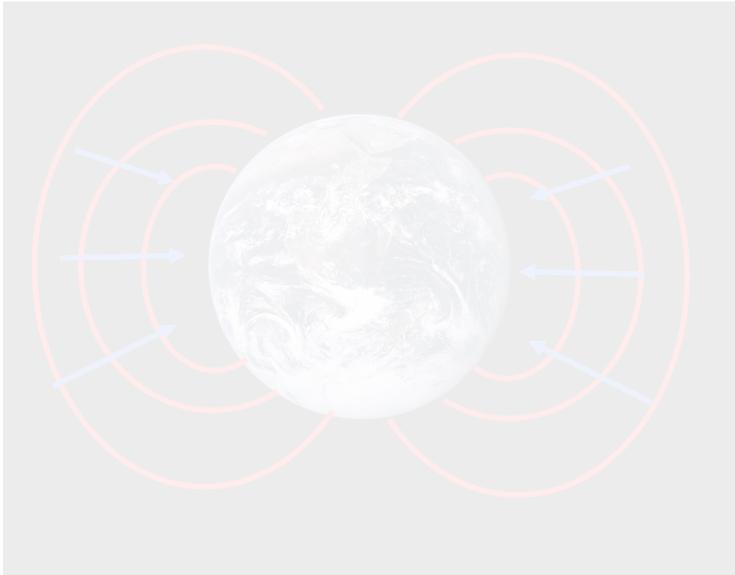


Problem of electron transport

- In many systems, electron current across confining magnetic field orders of magnitude higher than predicted from classical, i.e. collisional theory
- We do not know what drives this cross-field flow
- Precludes fully predictive modeling



Anomalous Cross-Field Transport Occurs in Many Plasma Systems

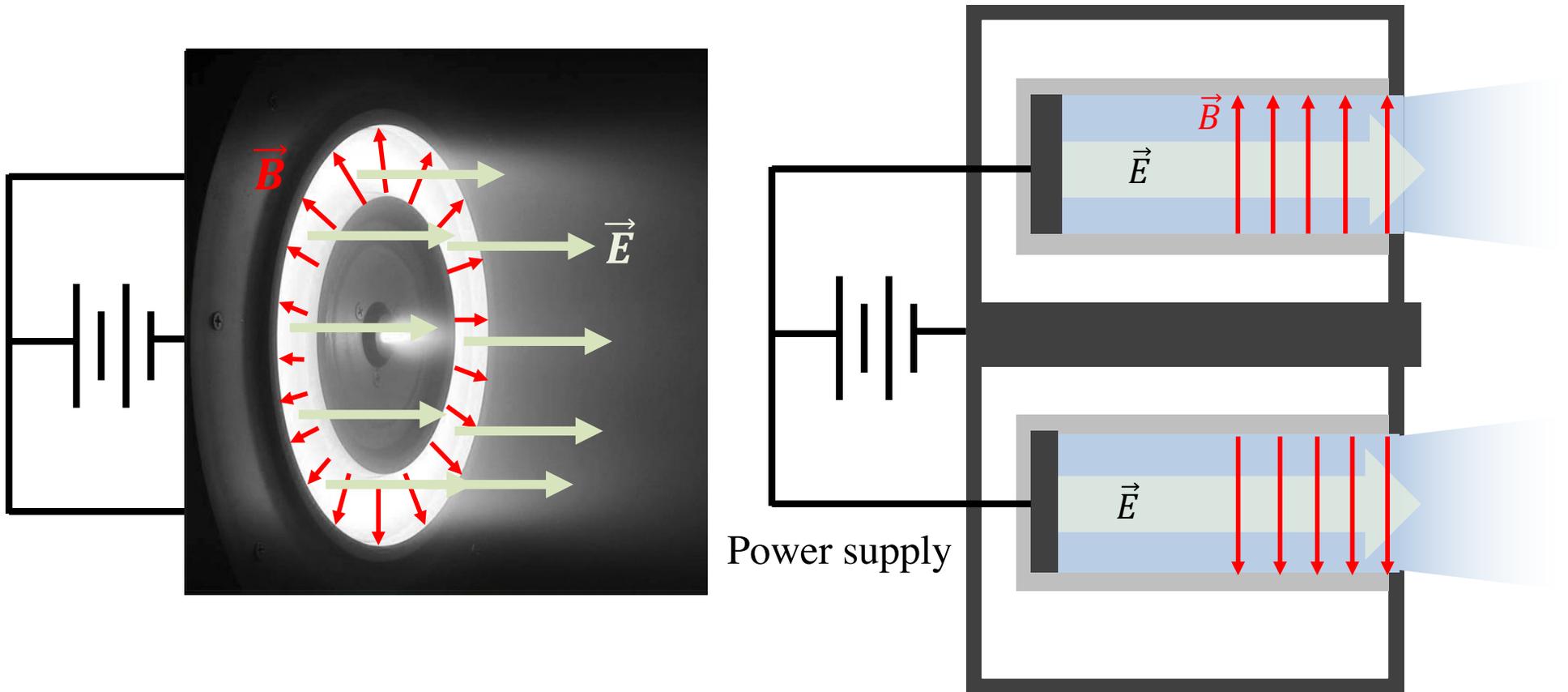


Plasma systems examined today

- Unmagnetized ions ($L_S \gg r_{ci}$)
- Low temperature electrons ($T_e < 70\text{eV}$)
- Examples:
 - Hall effect thrusters
 - Penning discharges
 - Some magnetic nozzles



Cross-field Transport in Partially Magnetized Plasmas



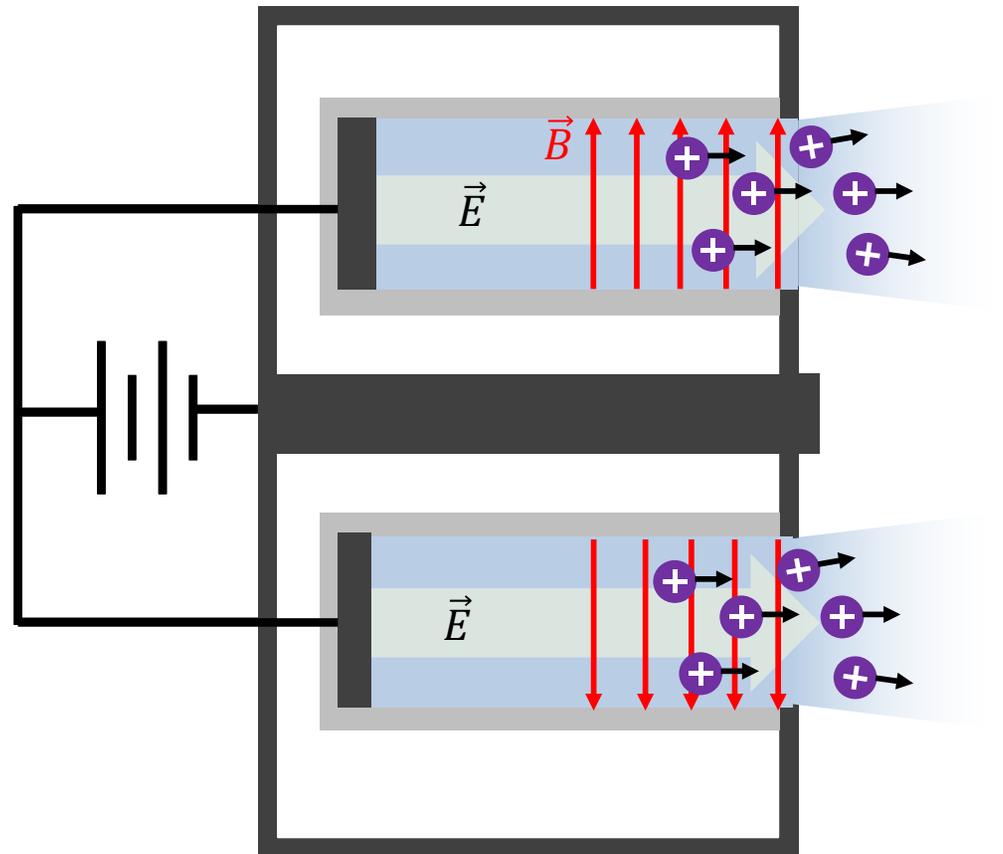
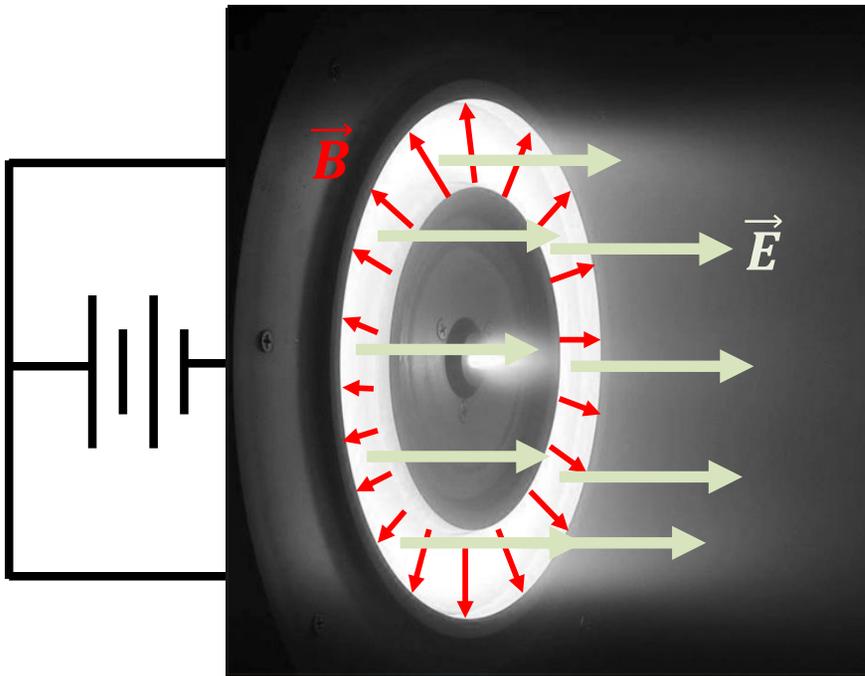


Cross-field Transport in Partially Magnetized Plasmas

Unmagnetized ions

$$\vec{V}_i = \frac{q}{m_i} \int \vec{E} dl$$

Ions follow the applied electric field



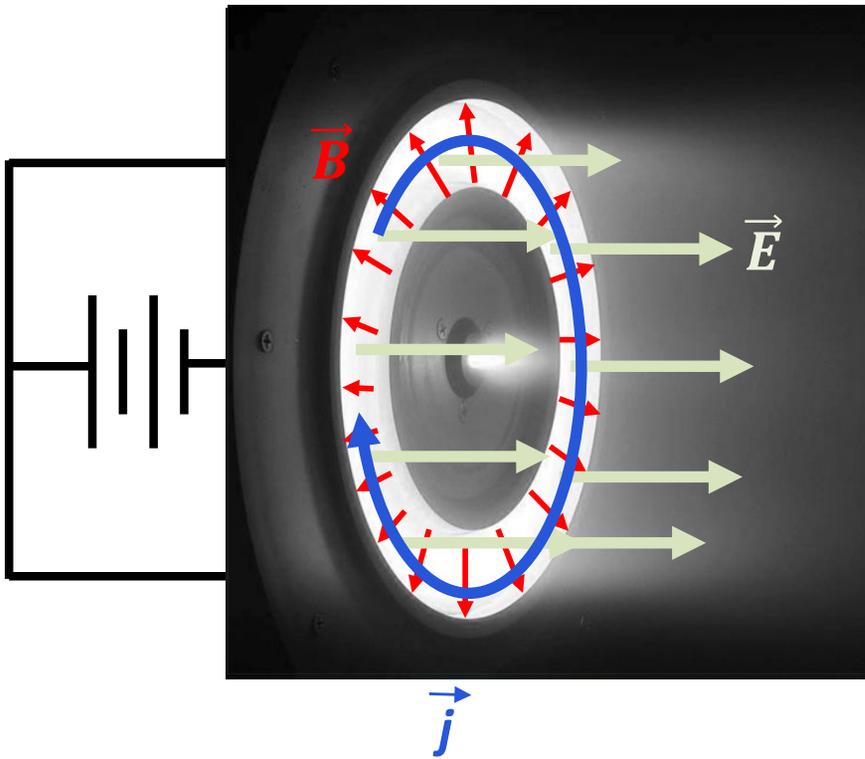


Cross-field Transport in Partially Magnetized Plasmas

Case 1: no electron collisions

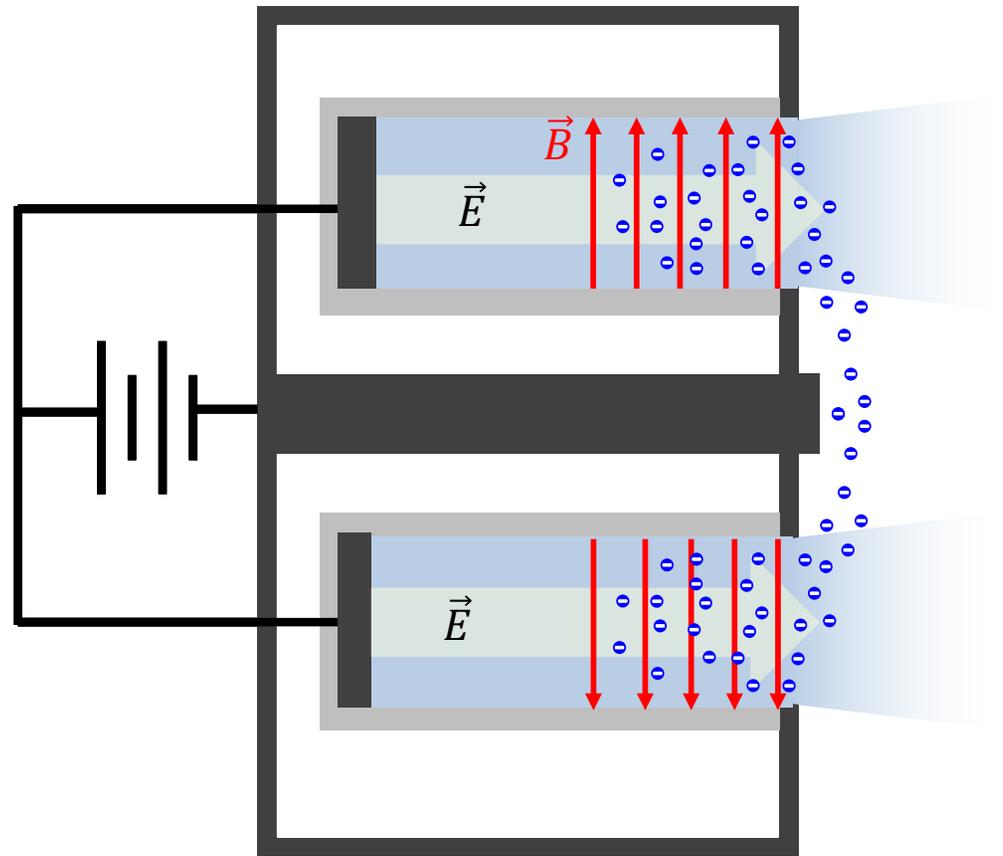
$$j_{\vec{E} \times \vec{B}} = -qn \frac{E}{B}$$

Electrons caught in $E \times B$ drift



$$j_{\vec{E}} = 0$$

No cross-field current





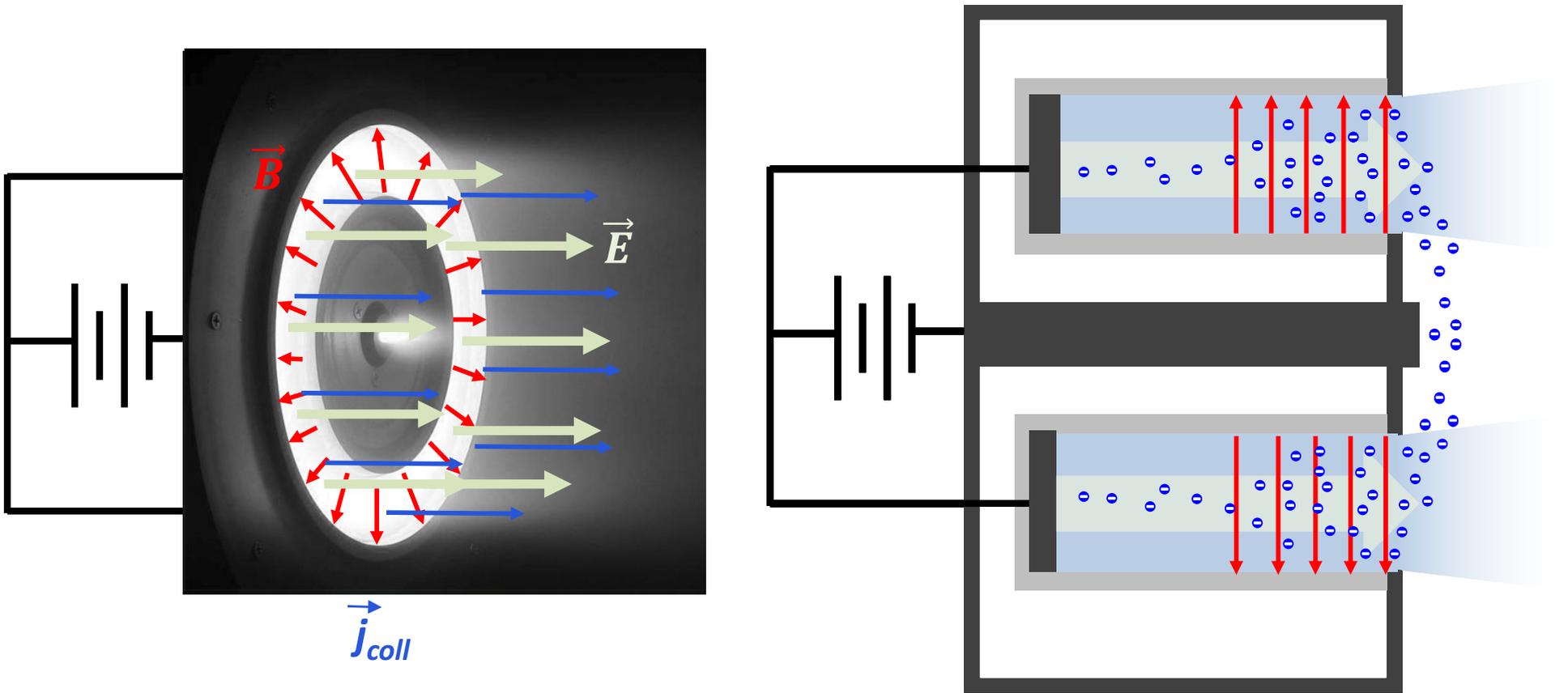
Cross-field Transport in Partially Magnetized Plasmas

Case 2: Allow for classical electron collisions

$$\Omega_e = \frac{\text{cyclotron frequency}}{\text{collision frequency}}$$

η = resistivity

$$j_{\vec{E}(coll.)} = E \frac{1}{\eta(1 + \Omega_e^2)}$$



Collisions allow for some electron current to cross field lines

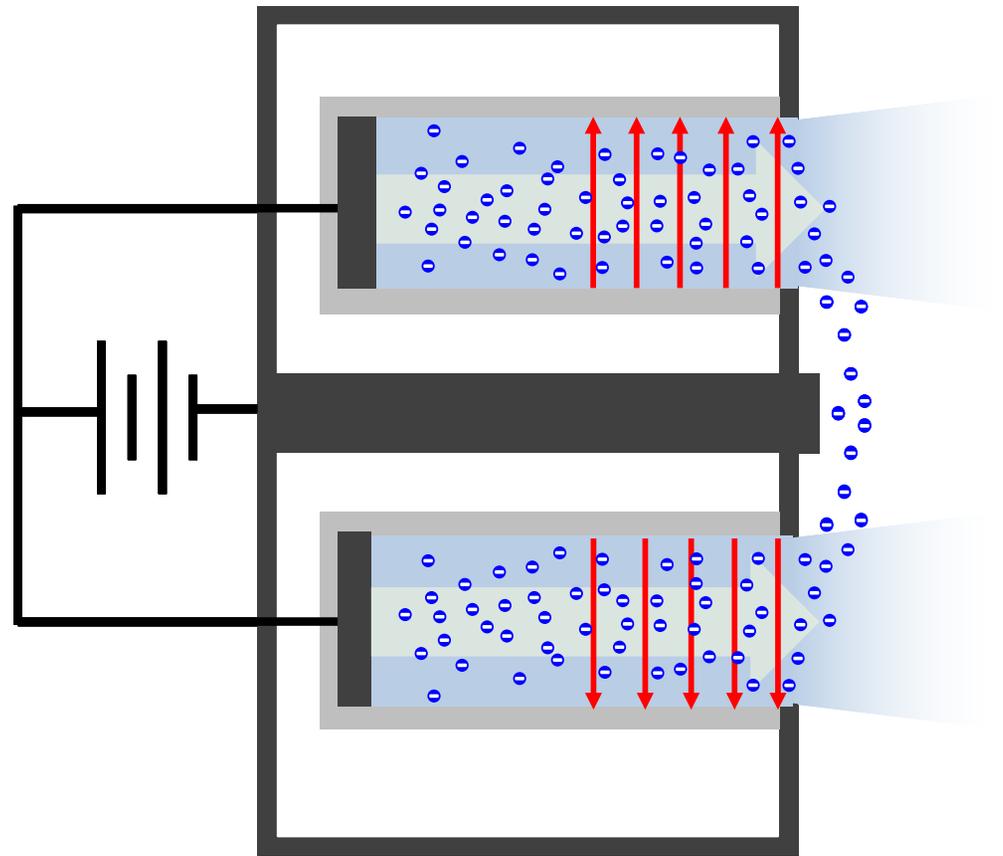
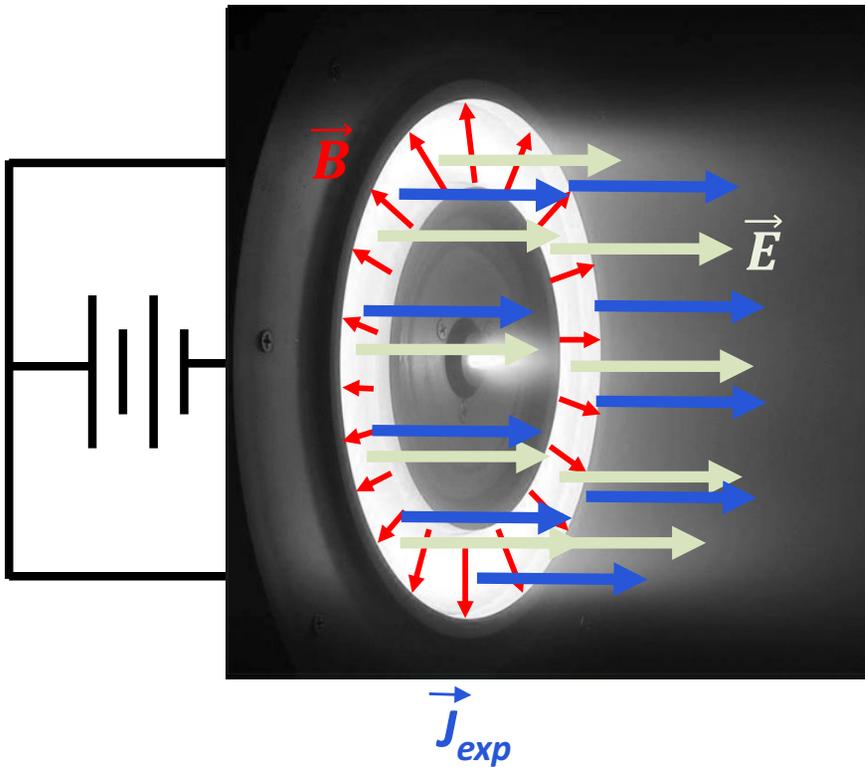


Cross-field Transport in Partially Magnetized Plasmas

Case 3: Experimental observations

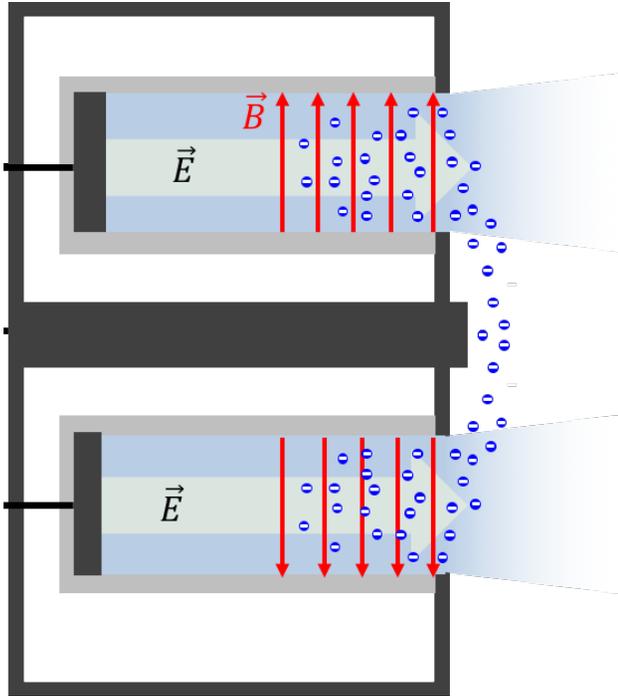
$$j_{\vec{E}(\text{exp.})} \gg j_{\vec{E}(\text{coll.})}$$

Experimental measurements show cross-field current orders of magnitude higher than transport induced by collisions

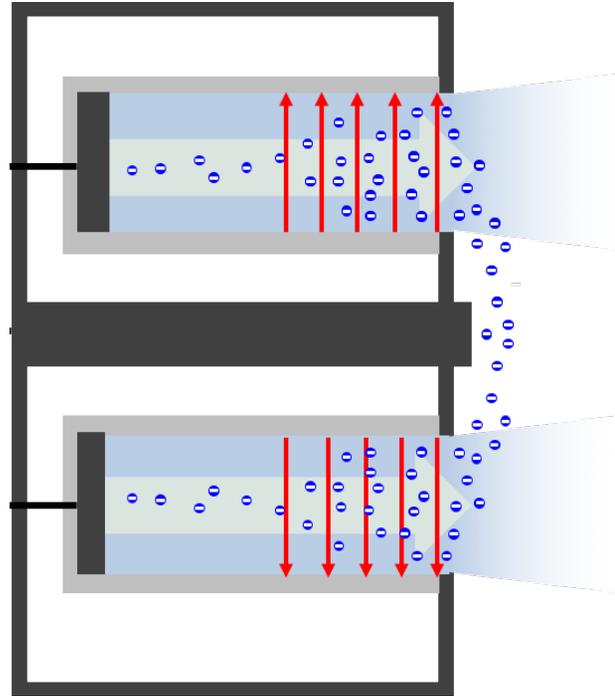




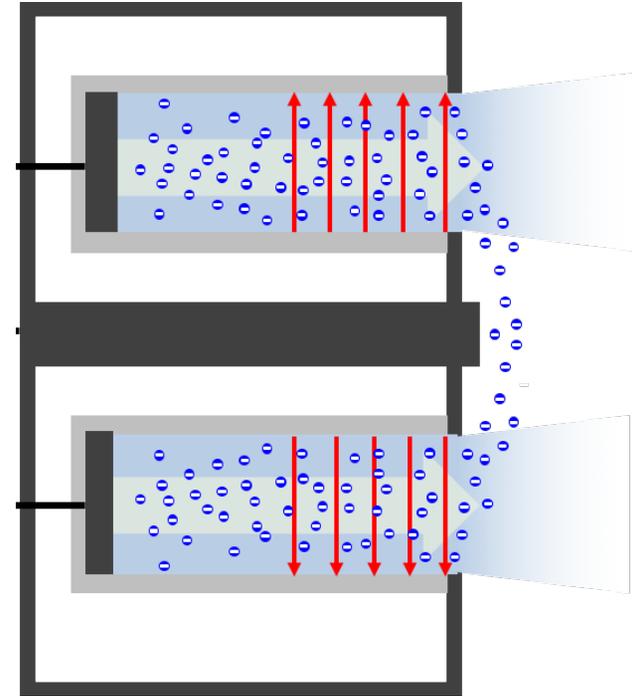
Cross-field Transport in Partially Magnetized Plasmas



Ideal (collisionless)



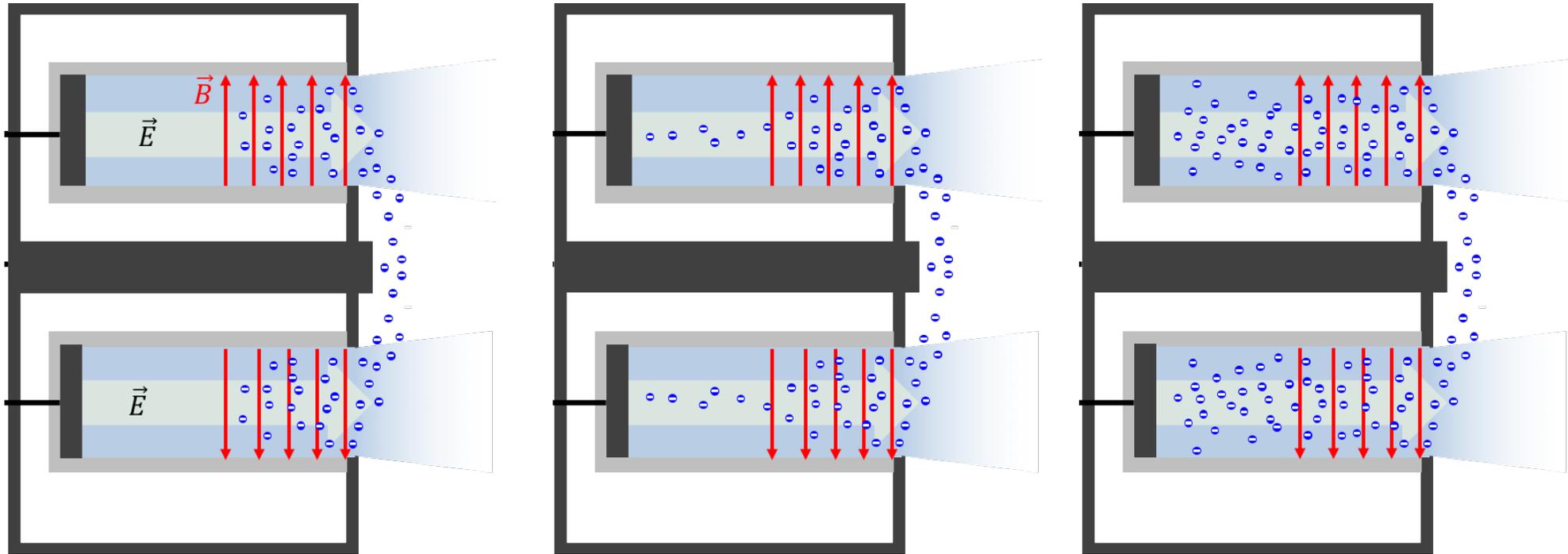
Classical transport from
particle collisions



Experimentally-observed,
anomalous transport



Cross-field Transport in Partially Magnetized Plasmas



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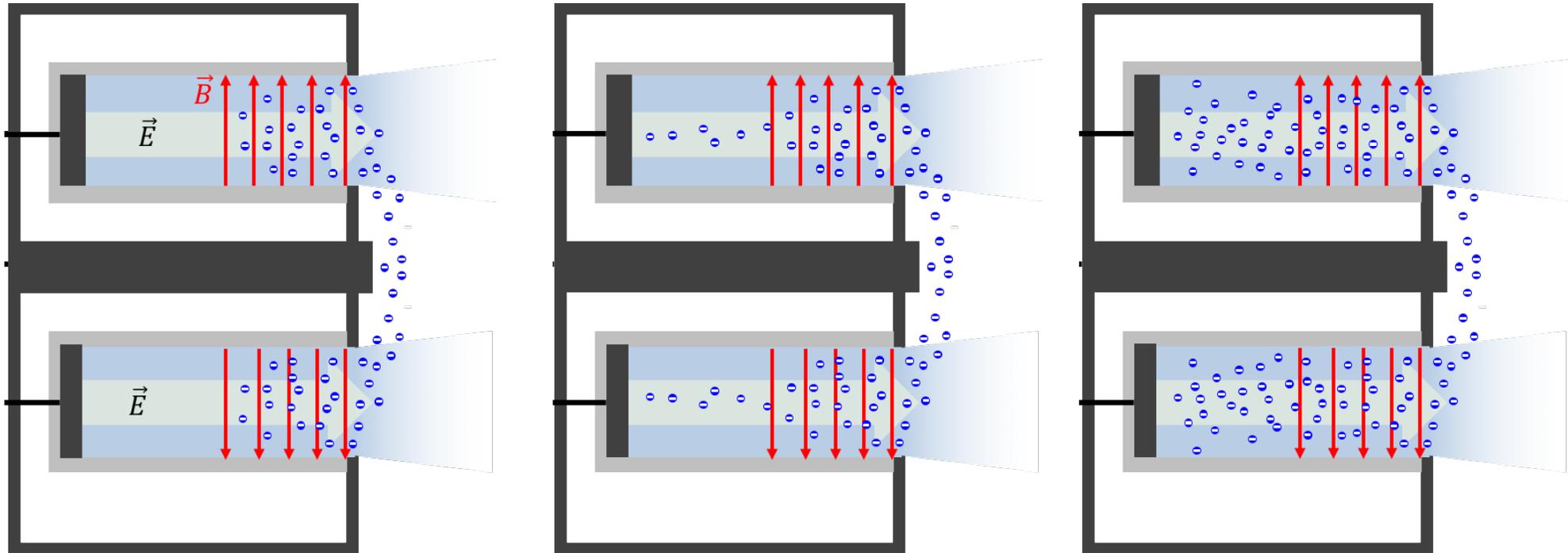
Experimentally-observed, anomalous transport

Adversely impacts efficiency

Precludes predictive models



Cross-field Transport in Partially Magnetized Plasmas



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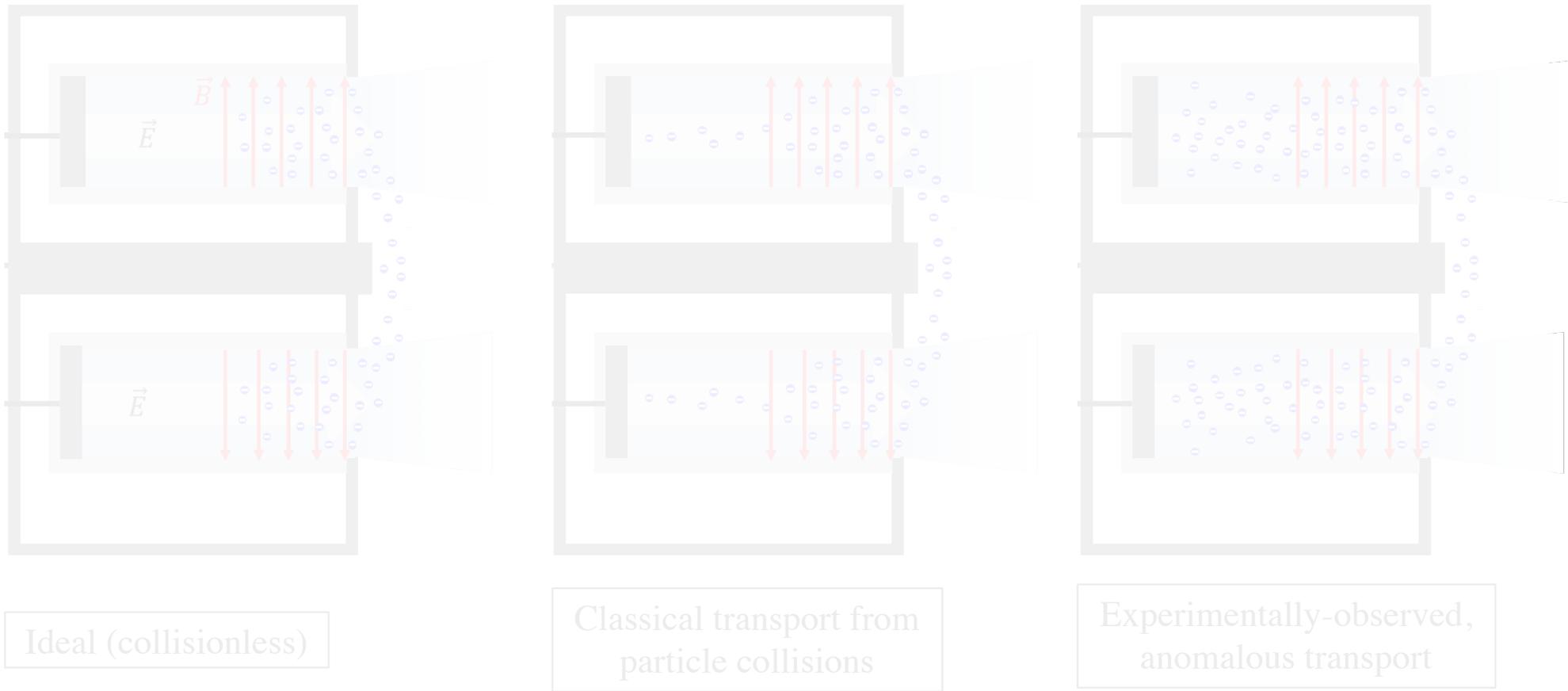
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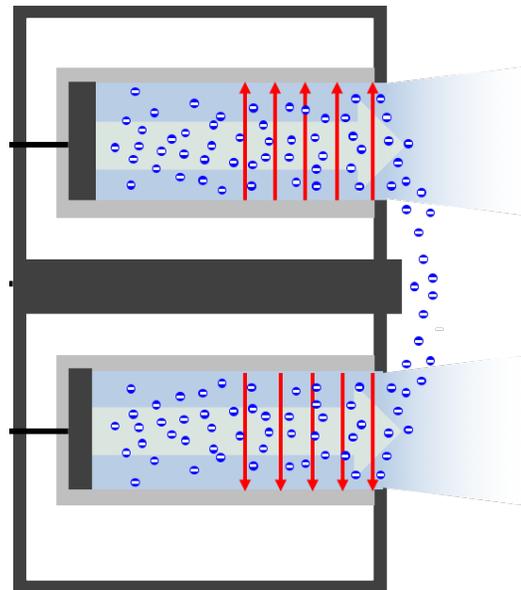
What drives this cross-field transport?



What anomalous force drives the transport?

Bohm Diffusion

Wall Interactions



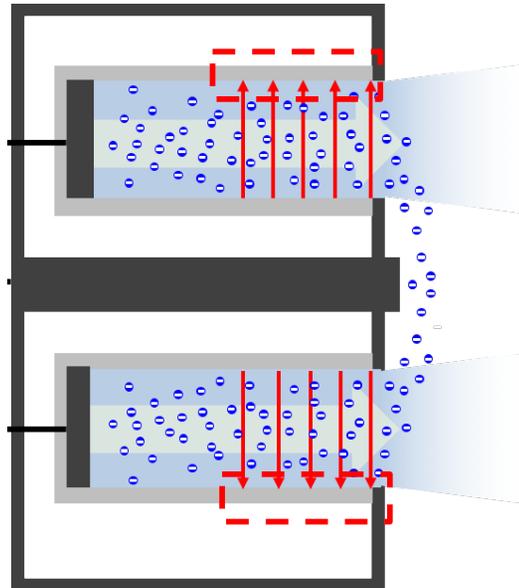
Instabilities



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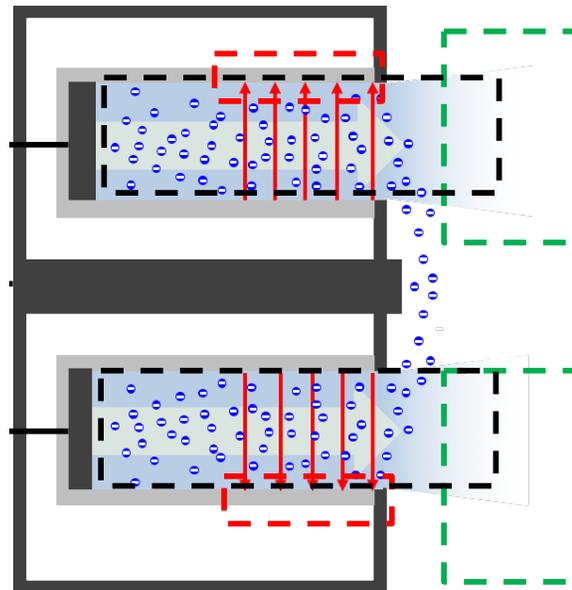
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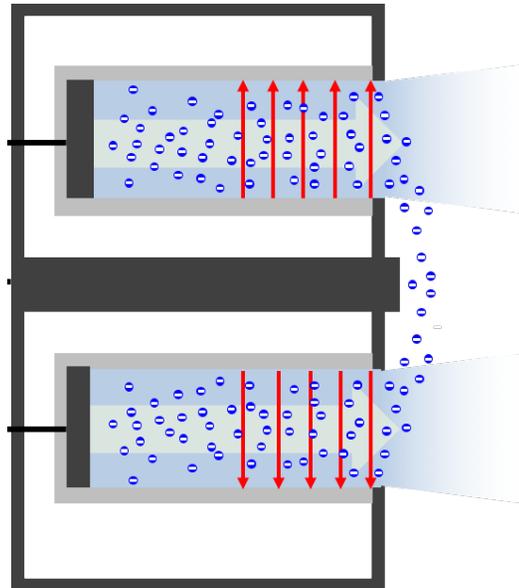
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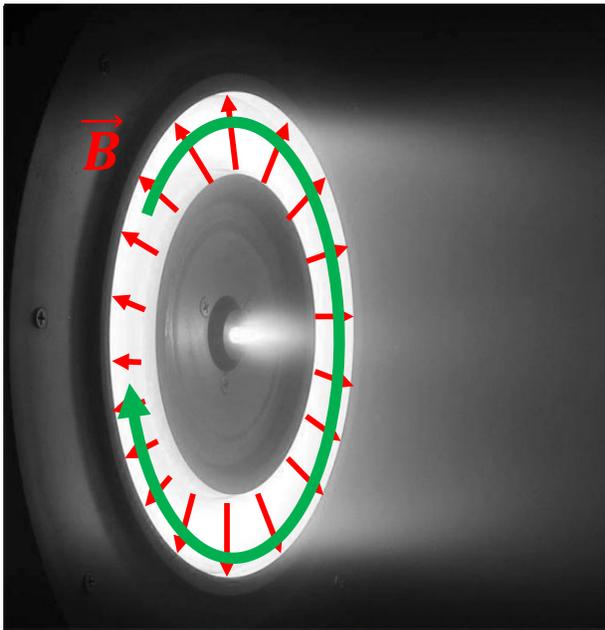
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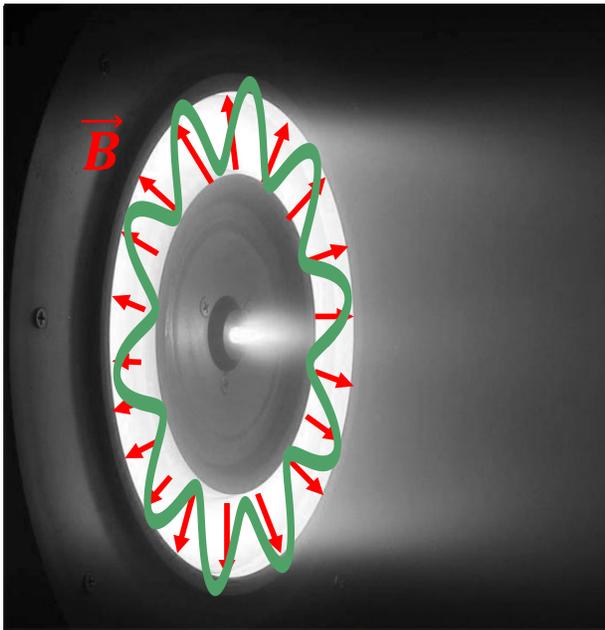


How can oscillations drive transport?



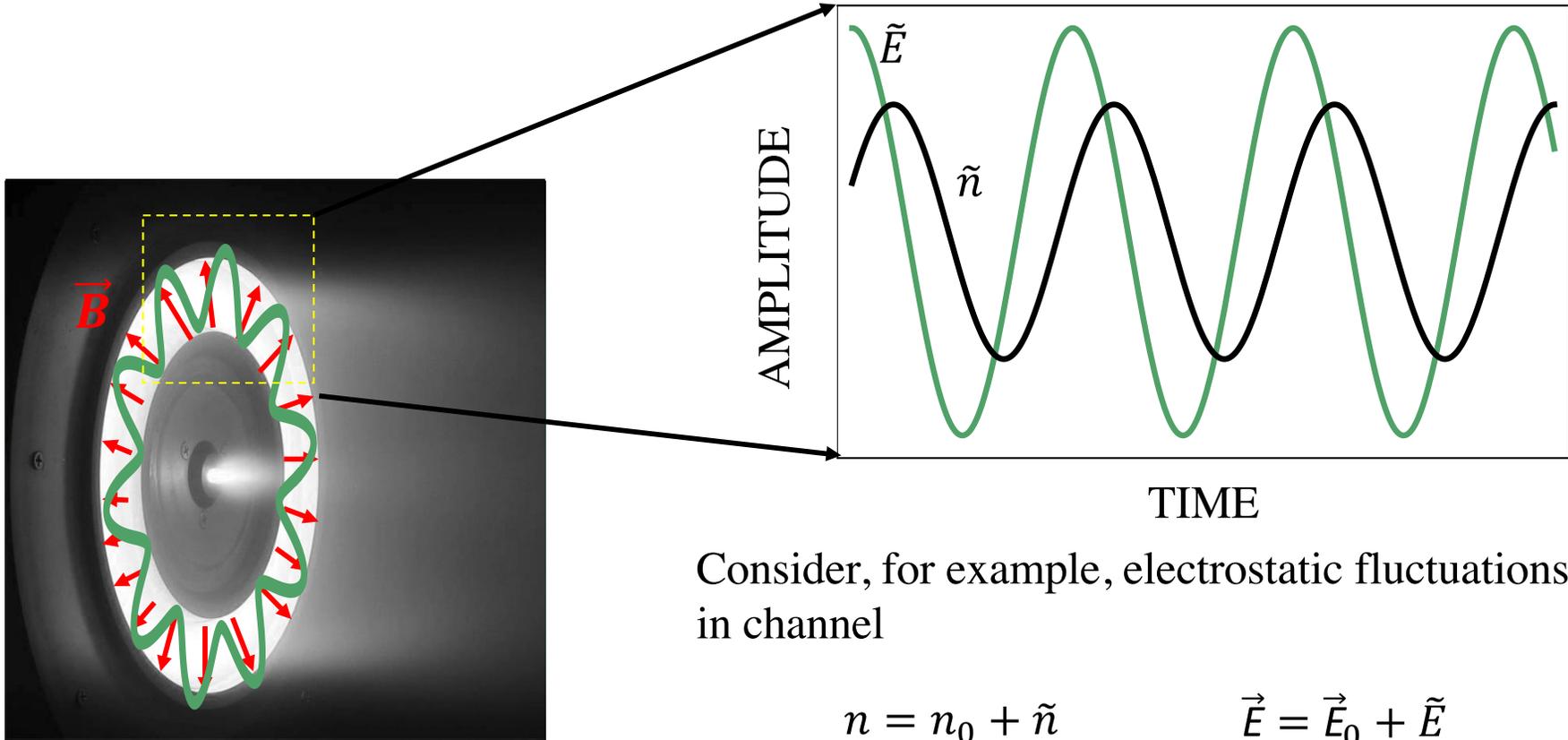


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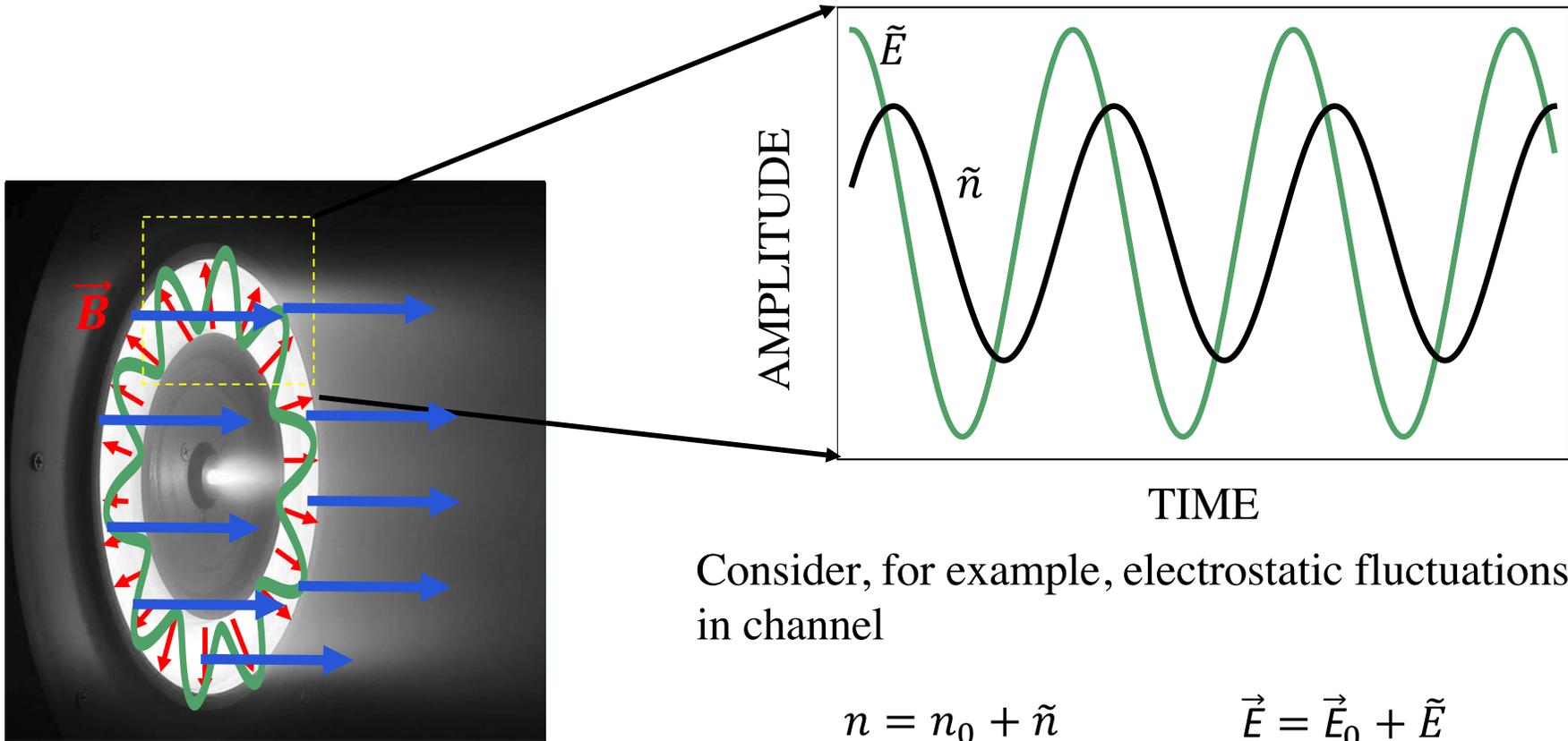
Consider, for example, electrostatic fluctuations in channel

$$n = n_0 + \tilde{n}$$

$$\vec{E} = \vec{E}_0 + \tilde{E}$$



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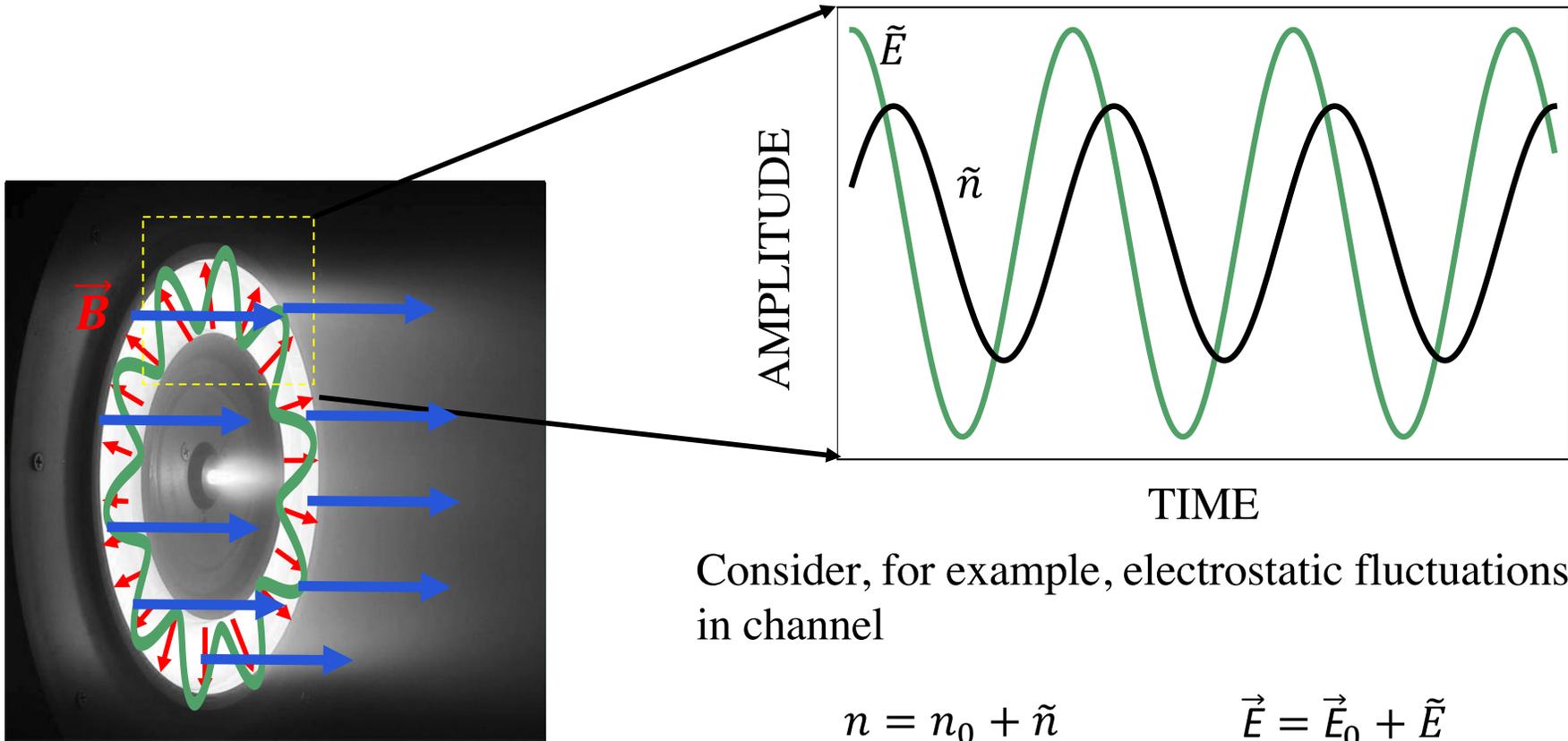
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Ohm's Law in limit of $\Omega_e \gg 1$

$$j_{\vec{E}} = \frac{q}{B} \left\langle \tilde{E}_{AN(\vec{E} \times \vec{B})} \tilde{n} \right\rangle$$



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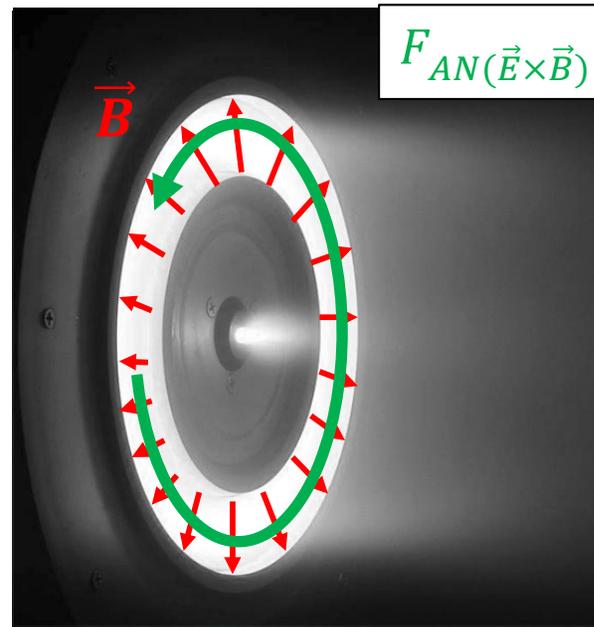
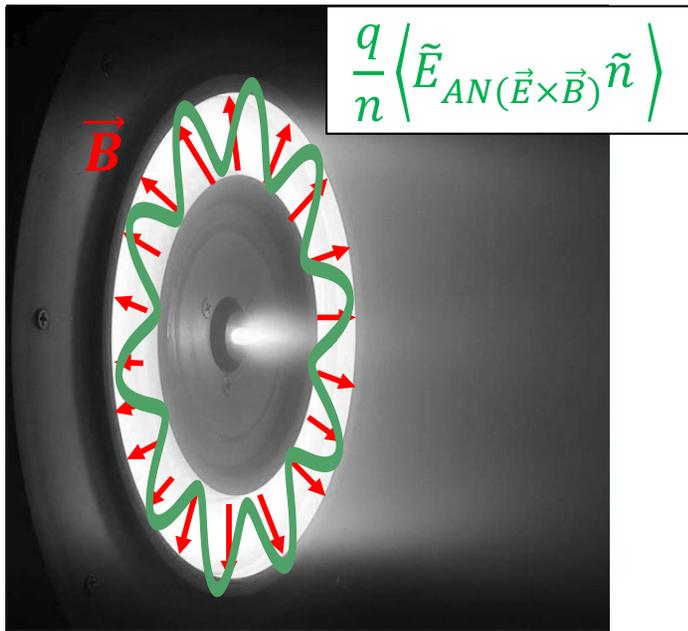
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Fluctuations in electric field and density in Hall direction can lead to cross-field transport



How can oscillations drive transport?



Physical interpretation: Electric field from oscillation in **Hall direction** provides anomalous “force” necessary to drive cross-field current

$$j_{\vec{E}} = \frac{q}{B} \langle \tilde{E}_{AN(\vec{E} \times \vec{B})} \tilde{n} \rangle$$

$$j_{\vec{E}} = n \frac{F_{AN(\vec{E} \times \vec{B})}}{B}$$



How can oscillations drive transport?

There are many types of oscillations in Hall thrusters:
What plasma modes are present in the accelerator that can provide this force?



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Plasma oscillations in Hall thrusters

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Electric Propulsion and Plasma Dynamics Laboratory, Princeton University, Princeton, New Jersey 08540

(Received 27 September 2000; accepted 14 November 2000)

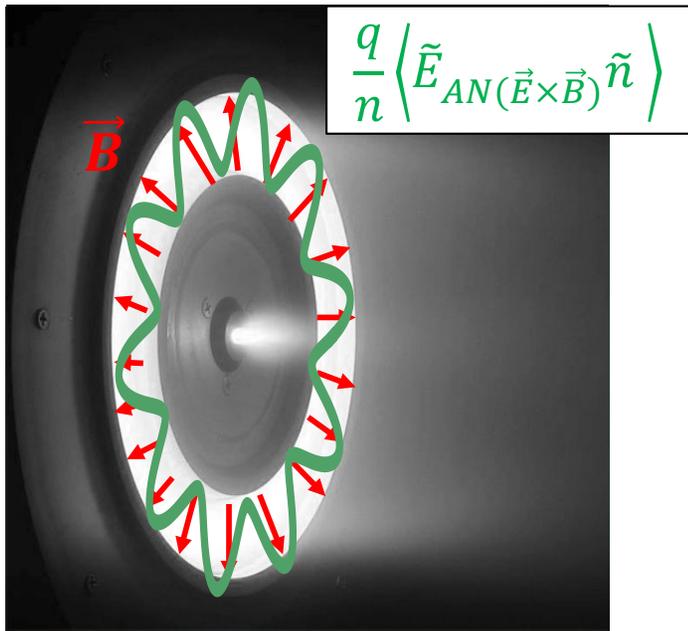
The nature of oscillations in the 1 kHz–60 MHz frequency range that have been observed during operation of Hall thrusters is quantitatively discussed. Contours of various plasma parameters measured inside the accelerating channel of a typical Hall thruster are used to evaluate the various stability criteria and dispersion relations of oscillations that are suspected to occur. A band by band up-to-date overview of the oscillations is carried out with a description of their observed behavior and a discussion of their nature and dependencies through comparison of the calculated contours to reported observations. The discussion encompasses the excitation of low frequency azimuthal drift waves that can form a rotating spoke, axially propagating “transit-time” oscillations, azimuthal drift waves, ionization instability-type waves, and wave emission peculiar to weakly ionized inhomogeneous plasmas in crossed electric and magnetic fields. © 2001 American Institute of Physics. [DOI: 10.1063/1.1354644]

$$\vec{j}_{\vec{E}} = \frac{q}{B} \left\langle \vec{E}_{AN(\vec{E} \times \vec{B})} \tilde{n} \right\rangle$$

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Many possible instabilities with component in Hall direction

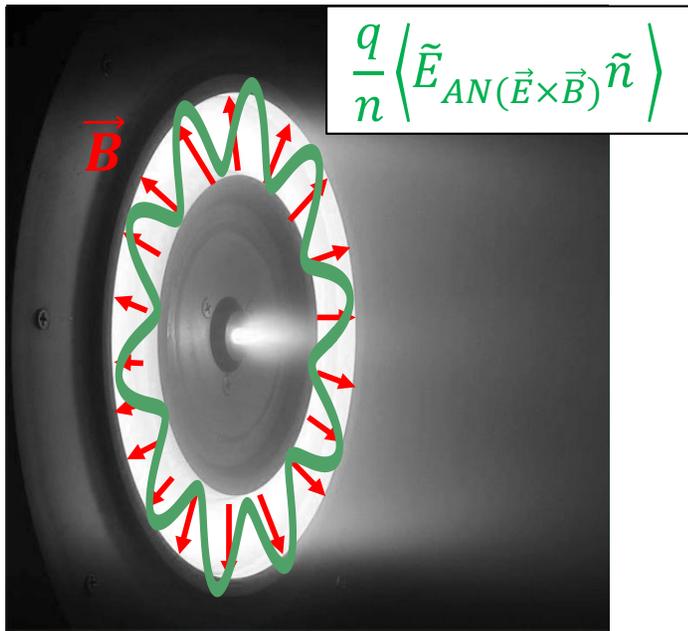


- Efforts have focused on identifying modes and their relative contributions to cross-field transport:
 - Resistive drift-waves
 - Spoke modes
 - Modified Simon-Hoh instability
 - Electron cyclotron drift instability (ECDI)
 - Ion acoustic-like modes
 - . . .

- Some to most of these instabilities actually exist in Hall effect thrusters and many are active areas of research



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How do we evaluate which modes are “important” and dominate cross-field transport?



Criteria to evaluate whether an instability drives the cross-field transport

1. Is there evidence that this type of mode should exist in the plasma?



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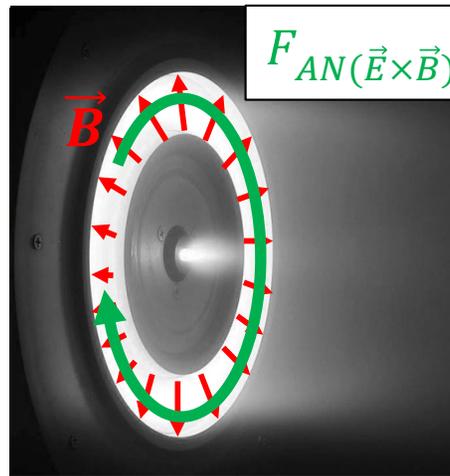
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4. Can the impact of this mode on transport be modeled self-consistently and is the modeled transport dominant?



Applying criteria to two popular modes for Hall thrusters

Bohm Diffusion

Wall Interactions



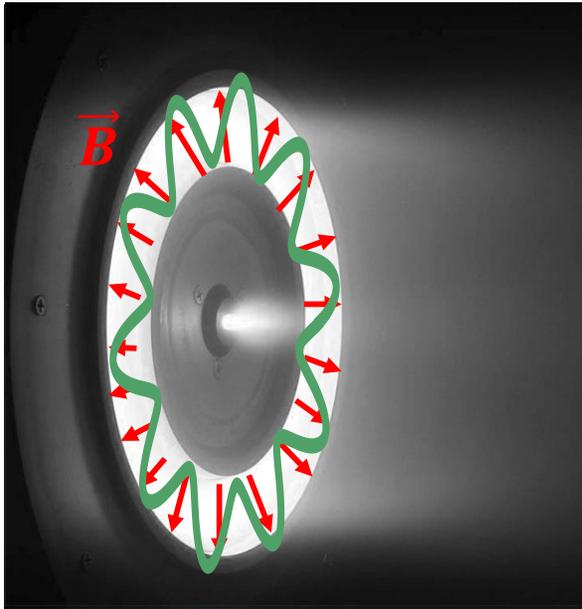
Instabilities

Large-scale
fluctuations

Micro
fluctuations



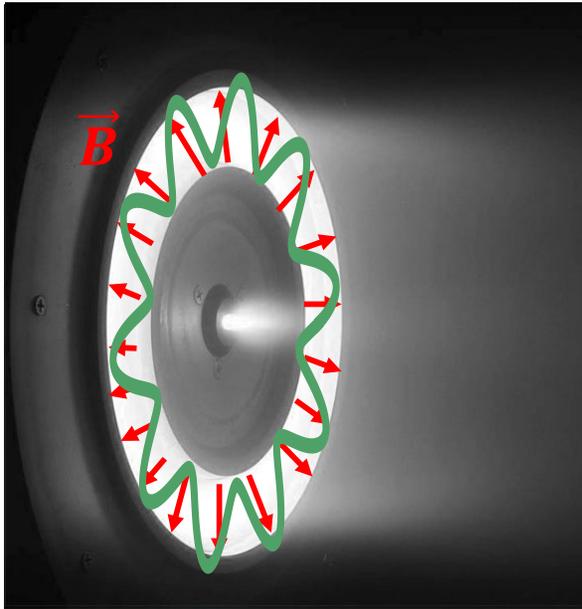
Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters





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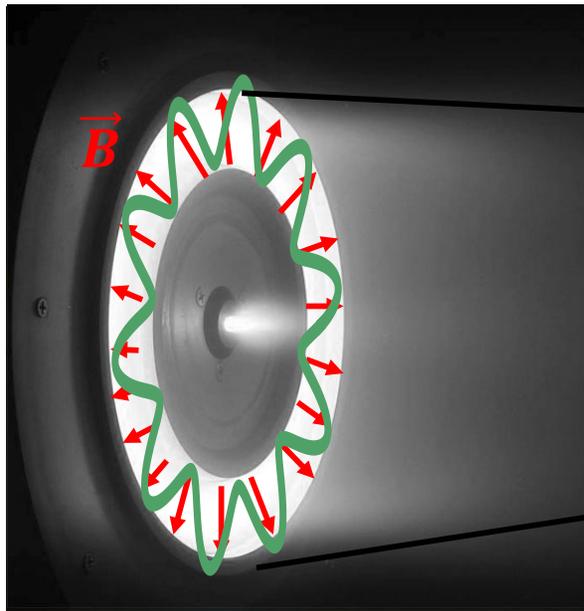
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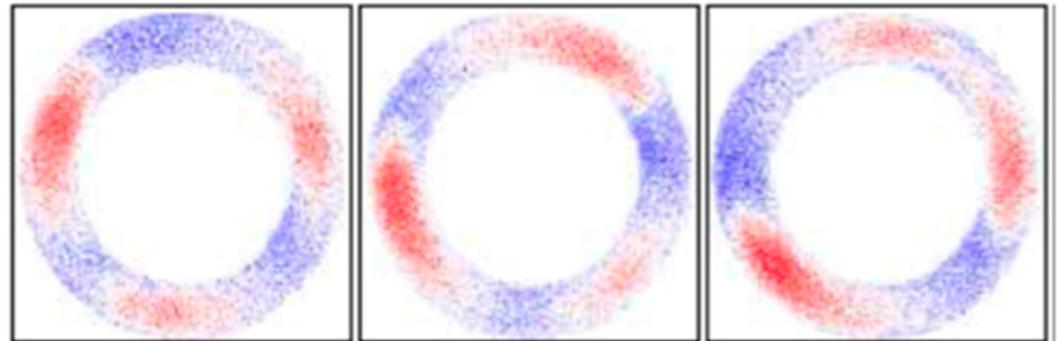


Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

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Images from high-speed video of light intensity fluctuations in Hall thruster*



t_0

t_1

t_2

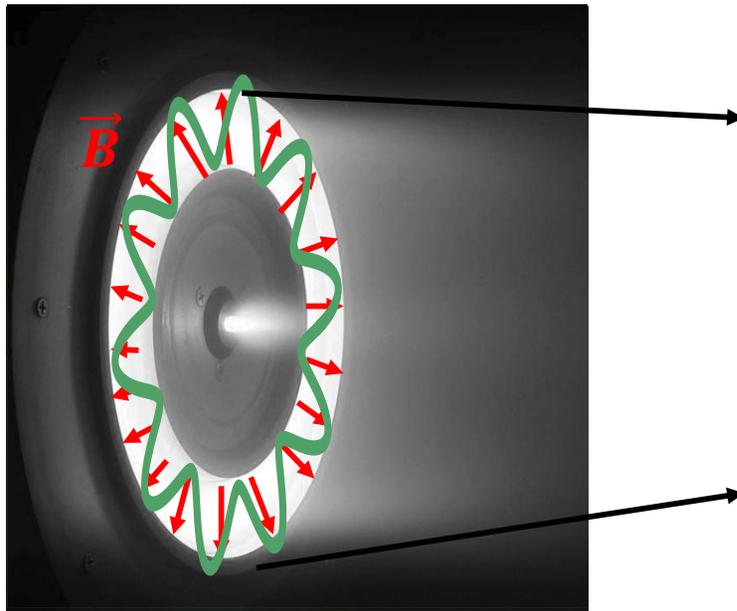
Rotating $m \geq 1$ modes at 20-50 kHz

*M. McDonald, C. Bellant, A. S. Brandon, and A. Gallimore, "Measurement of cross-field electron current in a Hall thruster due to rotating spoke instabilities," AIAA Paper No. 2011-5810, 2011.

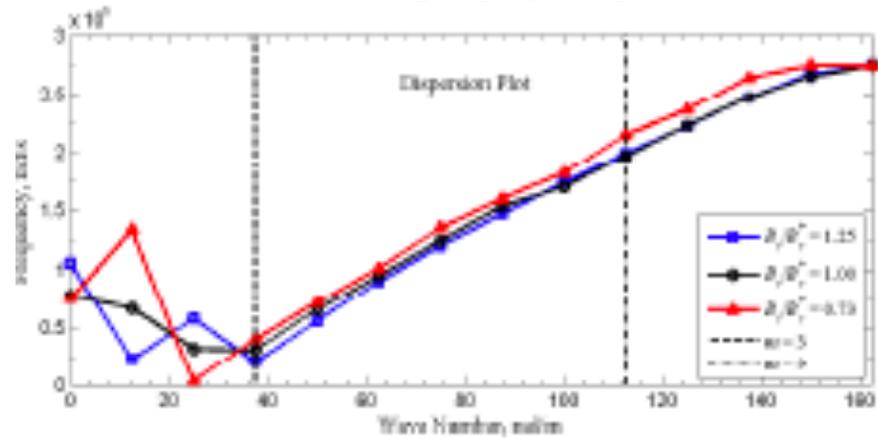


Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

1. Is there evidence that this mode should exist in the plasma?



Dispersion relation in the azimuthal direction for the propagating modes*



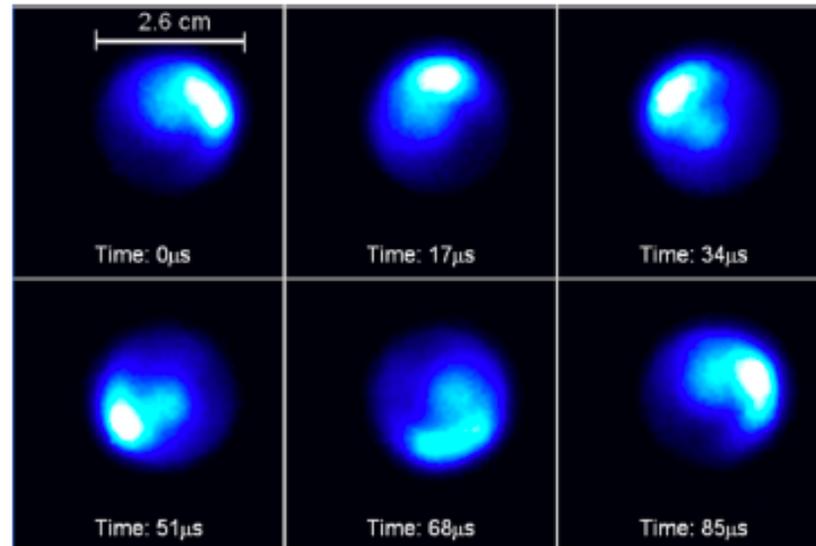
*M. Sekerak. Ph.D Thesis. University of Michigan. 2013.



Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

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Images from high-speed video of light intensity fluctuations in Cylindrical Hall thruster*



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*C. L. Ellison, Y. Raitses, and N. J. Fisch, “Cross-field electron transport induced by a rotating spoke in a cylindrical Hall thruster,” Phys. Plasmas 19, 013503 (2012).



Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

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Other probing studies

- C. S. Janes and R. S. Lowder, “Anomalous electron diffusion and ion acceleration in a low density plasma,” *Phys. Fluids* 9, 1115 (1966).
- E. Chesta, M. Lam, N. B. Meezan, D. Schmidt, and M. A. Cappelli, “A characterization of plasma fluctuations within a Hall discharge,” *IEEE Trans. Plasma Sci.* 29, 582 (2001).
- A. W. Smith and M. A. Cappelli, “Time and space-correlated plasma potential measurements in the near field of a coaxial Hall plasma discharge,” *Phys. Plasmas* 16, 073504 (2009).

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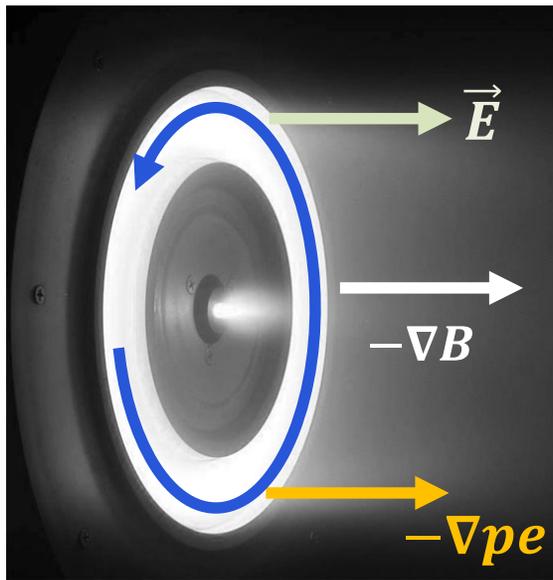
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What is the mode?



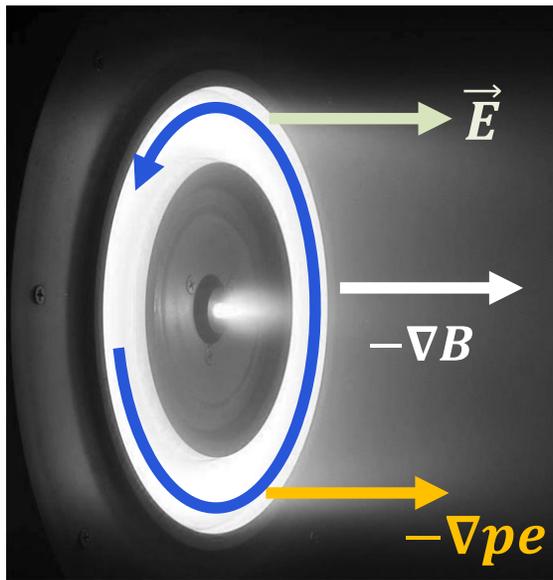
- Gradients in plasma act as free source of energy driving stabilizing electron drift in Hall direction
- A number of modes can result, but the nature of the mode in the thruster is still unclear



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Some potential candidates

- Modified Simon-Hoh instability*
- Ionization related mode**
- Energy transfer from kinetically-driven short wave length oscillations***

*W. Frias, A. I. Smolyakov, I. D. Kaganovich, and Y. Raitses, “Long wavelength gradient drift instability in Hall plasma devices. I. Fluid theory,” [Phys. Plasmas](#) 19, 072112 (2012).

**D. Escobar and E. Ahedo. “Low frequency azimuthal stability of the ionization region of the Hall thruster discharge. I. Local analysis.” [Phys. Plasmas](#) 21, 043505 (2014)

***S. Janhunen, A. Smolyakov, and O. Chapurin. “Non-linear structures and anomalous transport in partially magnetized plasma driven by the transverse current.”



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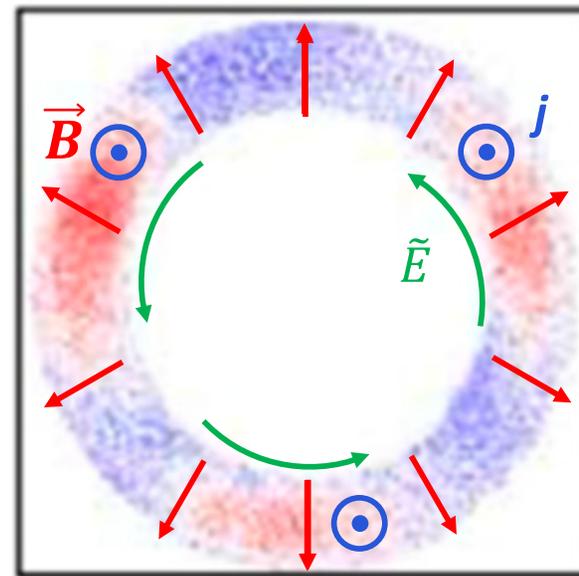
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How does it move electrons?



Electric field perturbations in Hall direction in phase with density fluctuations lead to cross field transport



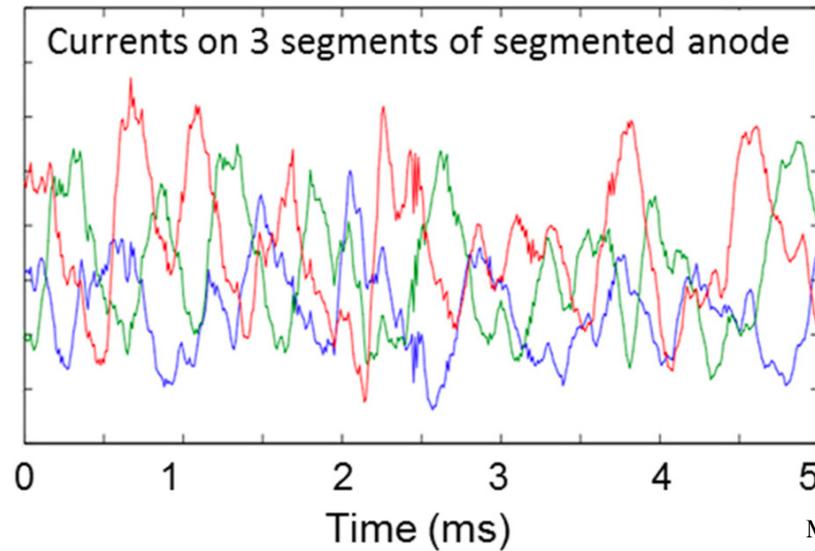
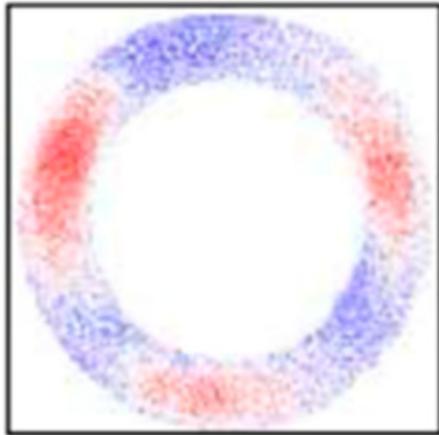
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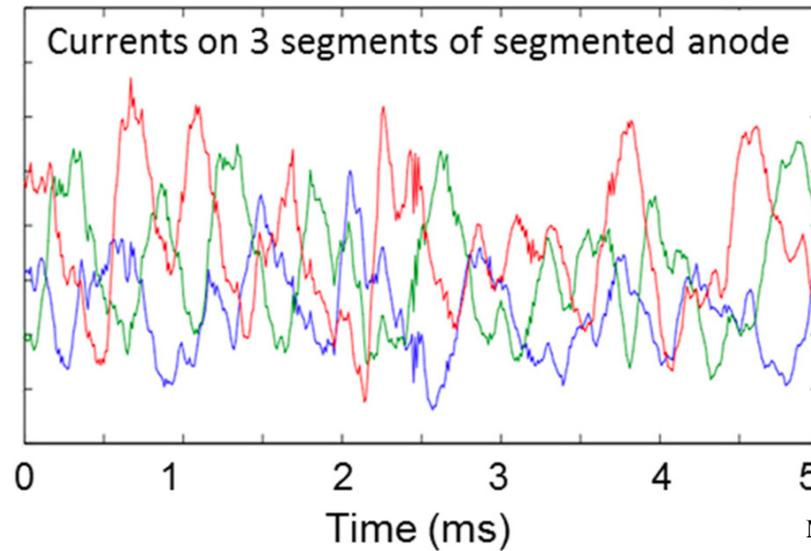
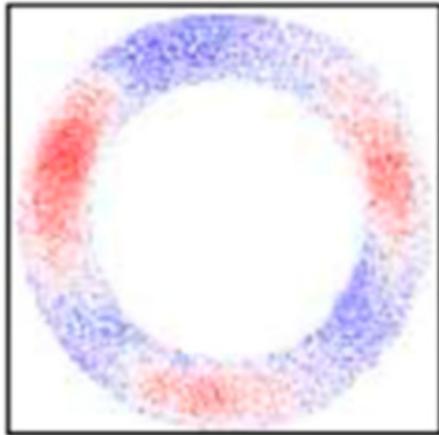


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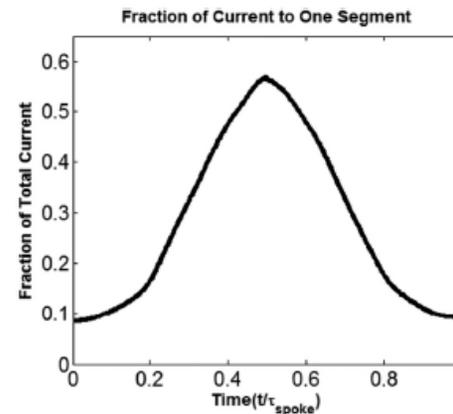
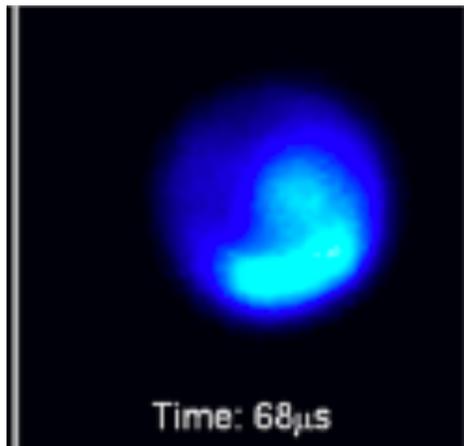


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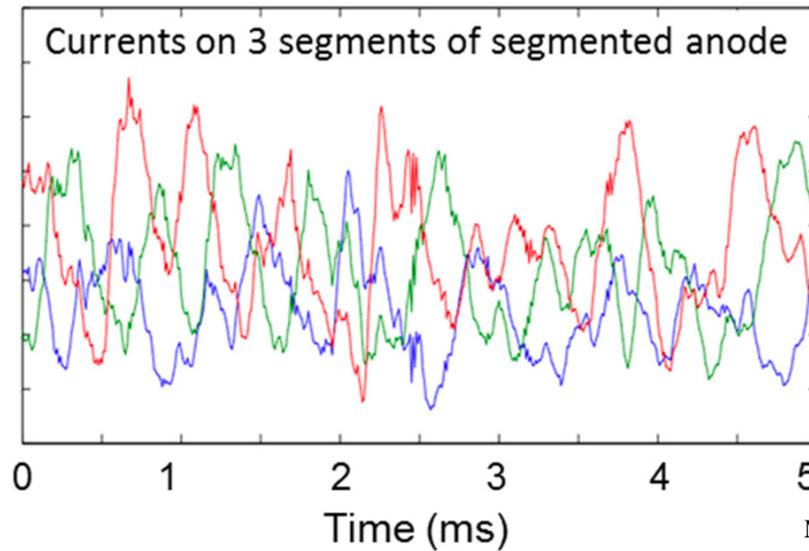
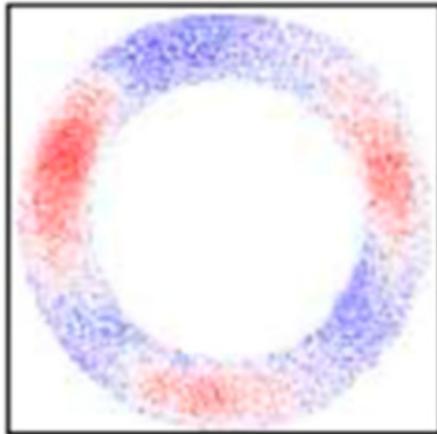


C. L. Ellison, Y. Raitses, and N. J. Fisch, Phys. Plasmas 19, 013503 (2012).

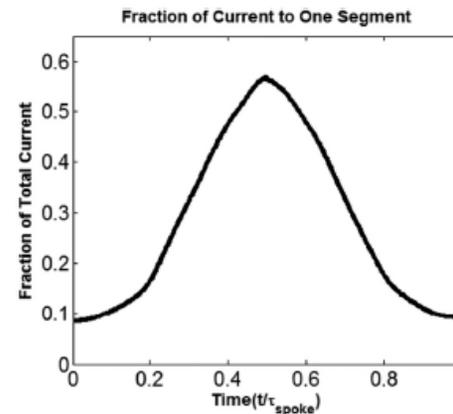
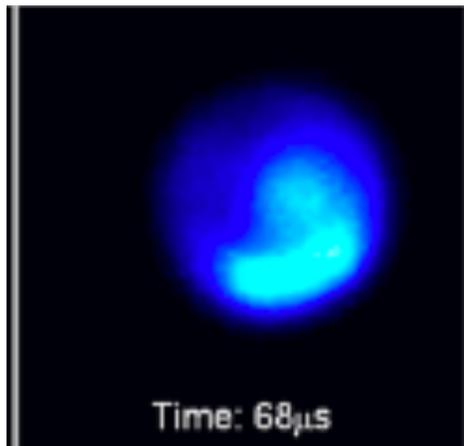


Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

3. Is there evidence that this mode is sufficiently strong to drive observed cross-field electron transport?



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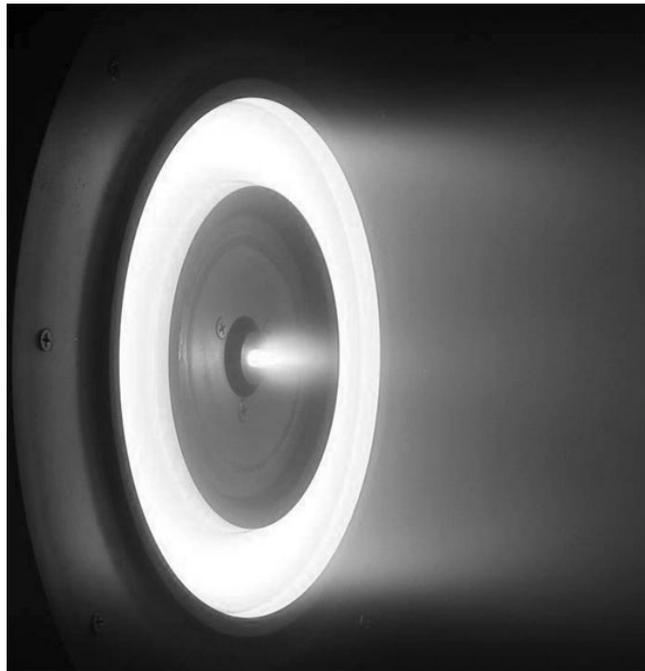
C. L. Ellison, Y. Raitses, and N. J. Fisch, Phys. Plasmas 19, 013503 (2012).

Both studies showed 50% of electron current is conducted by spoke across field lines, but there are caveats



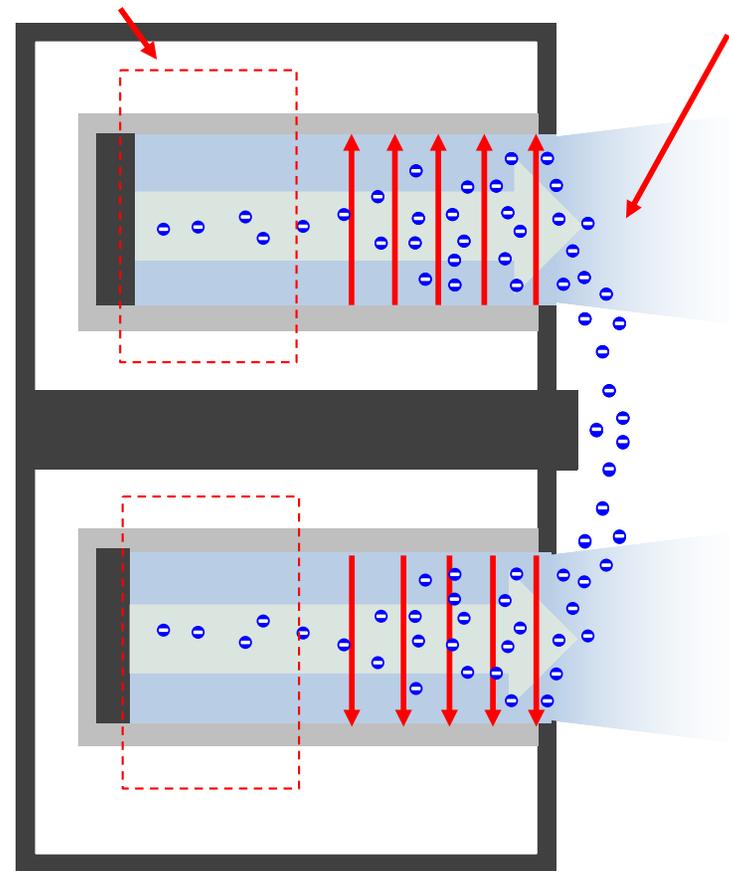
Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

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Spoke propagating

Maximum transport



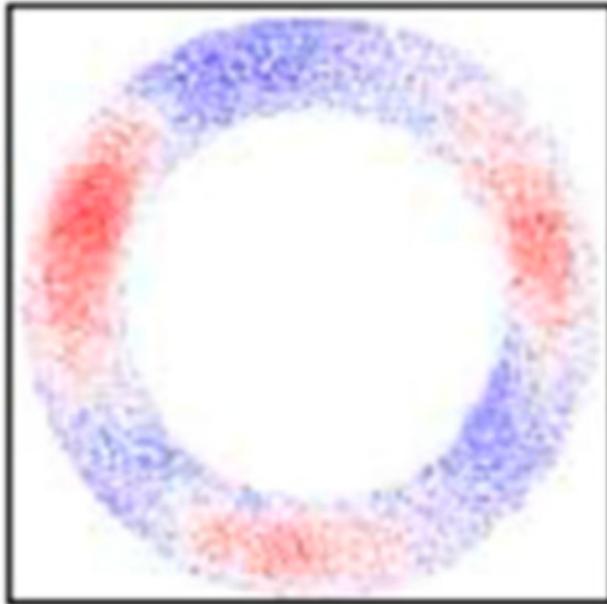
Some experimental evidence suggests spoke only exists near anode, but majority of anomalous diffusion is needed downstream



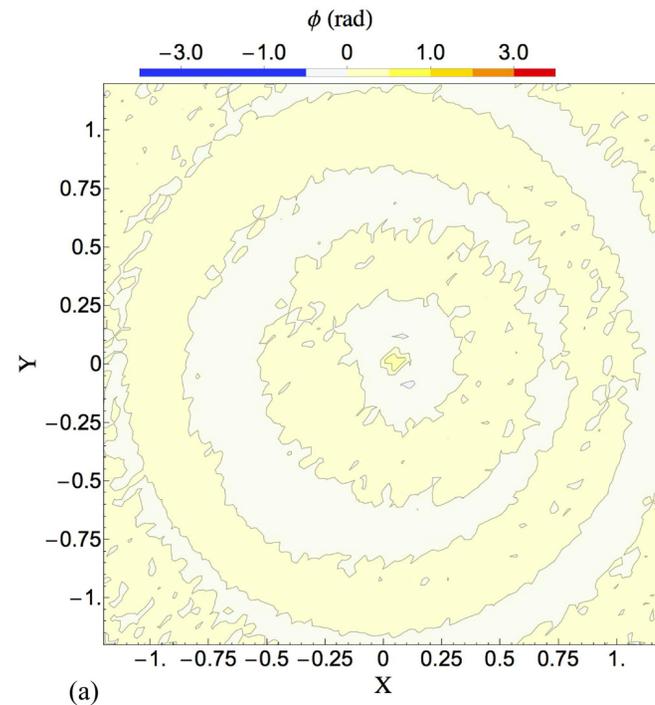
Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

3. Is there evidence that this mode is sufficiently strong to drive observed cross-field electron transport?

Traditional 6-kW thruster*



Magnetically shielded 6-kW thruster**



Spoke doesn't exist in all thrusters, but yet they operate in the same way

*M. McDonald, C. Bellant, A. S. Brandon, and A. Gallimore, AIAA2011-5810, 2011.

**B. Jorns and R. Hofer. "Plasma Oscillations in a 6-kW Magnetically Shielded Hall Thruster," Physics of Plasmas 21 , 053512 (2014);



Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

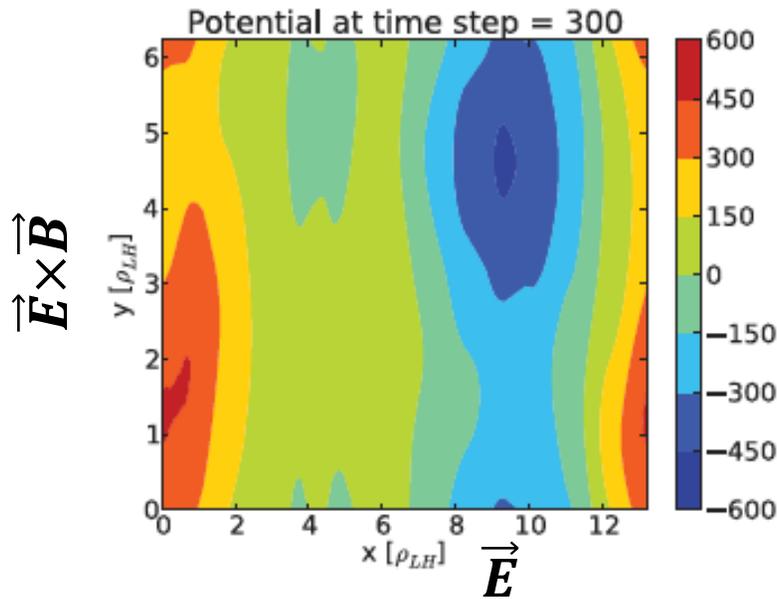
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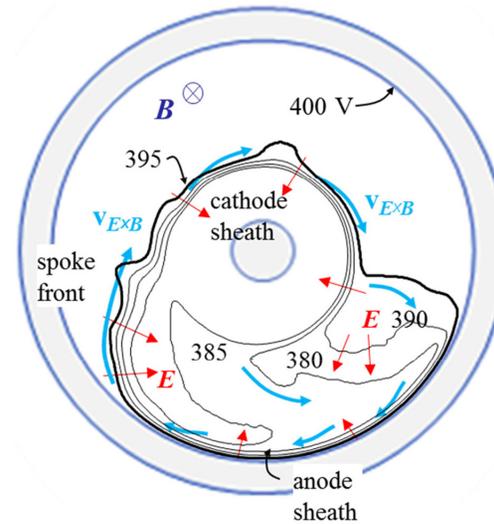
Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

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Nonlinear fluid simulations*



2D Particle-in-Cell**



*A. Smolyakov et al, “Fluid theory and simulations of instabilities, turbulent transport and coherent structures in partially-magnetized plasmas of ExB discharges,” Plasma Phys. Control. Fusion 59 (2017) 014041.

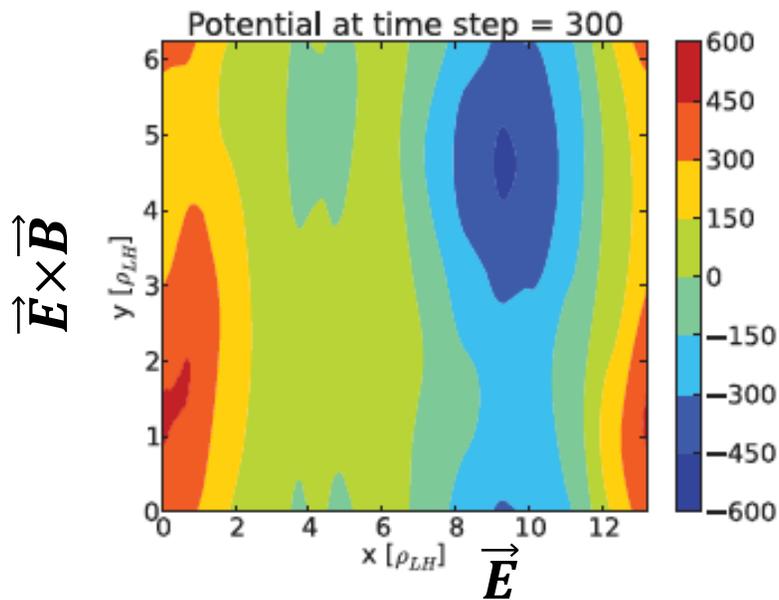
**J.P. Boeuf. “Rotating structures in low temperature magnetized plasmas—insight from particle simulations,” Frontiers in Physics, Vol. 2, No. 74, (2014)



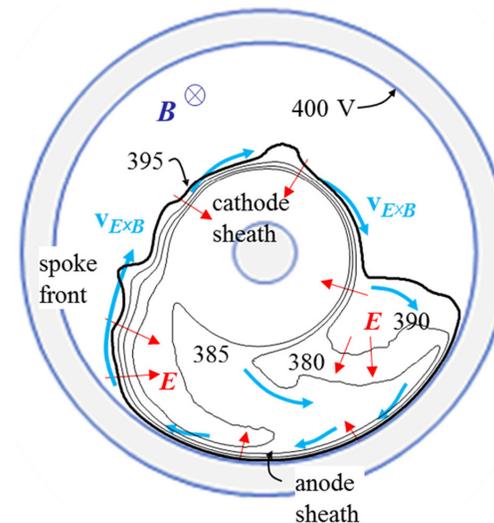
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Simulations to date show that modes can onset and contribute to cross-field transport

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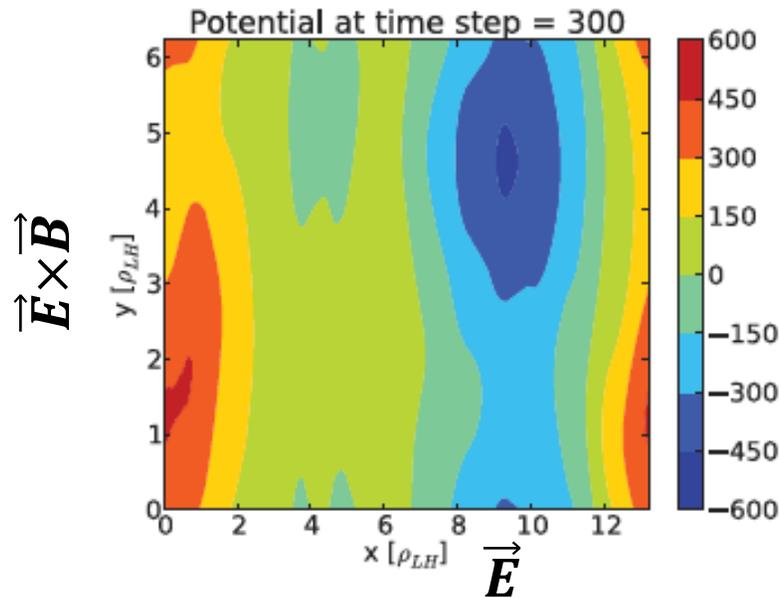
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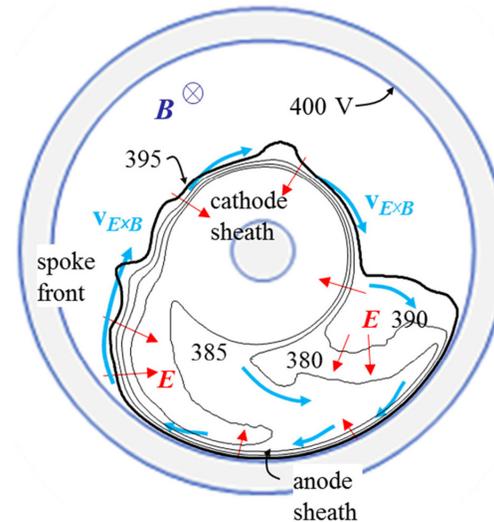
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Nonlinear fluid simulations*



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Simulations to date show that modes can onset and contribute to cross-field transport

- Not clear if cross-field current is “enough”
 - “. . .the mechanism for spoke formation is still not clear”*
 - Models do not yet have fidelity for capturing actual thruster geometry

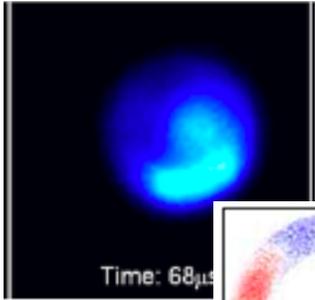
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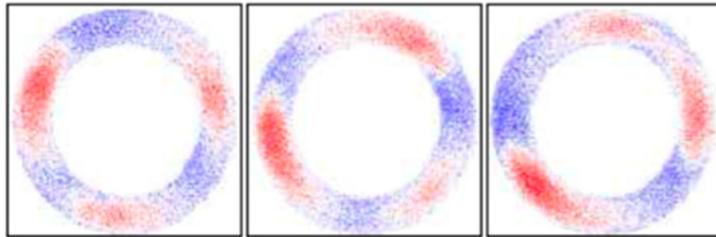


Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

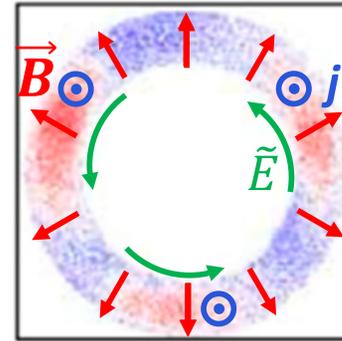
Is there evidence that this mode should exist in the system?



- Experimental ✓
- Numerical ✓
- Analytical ✓

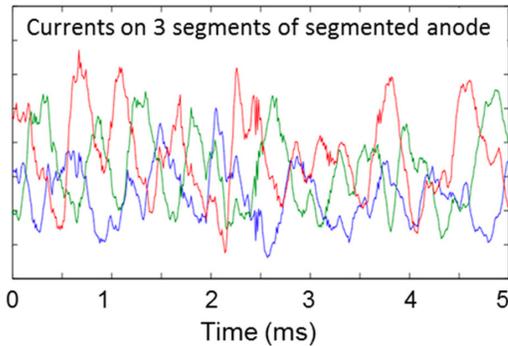


Is there a first-principles explanation for this mode and how it impacts electron transport?



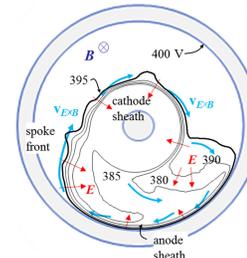
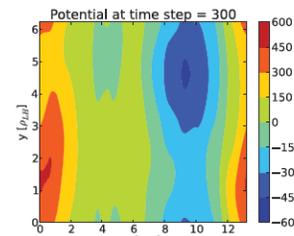
- Energy source from gradients ✓
- Transport due to electric field perturbations in Hall direction ✓

Is there evidence that this mode is sufficiently strong to drive observed cross-field electron transport?



- Direct measurements of cross-field current from this mode ✓
- However, current is not necessarily in right place (?)
- Mode not ubiquitous (?)

Can the impact of this mode on transport be modeled self-consistently and is the modeled transport dominant?

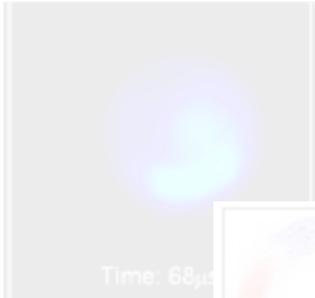


- Multiple hierarchies being explored that predict some level of transport ✓
- Models are still simplified geometries (?)
- Not clear if this is dominant effect (?)



Case Study 1: Long-wavelength, azimuthal structures in Hall thrusters

Is there evidence that this mode should exist in the system?



- Experimental ✓
- Numerical ✓
- Analytical ✓



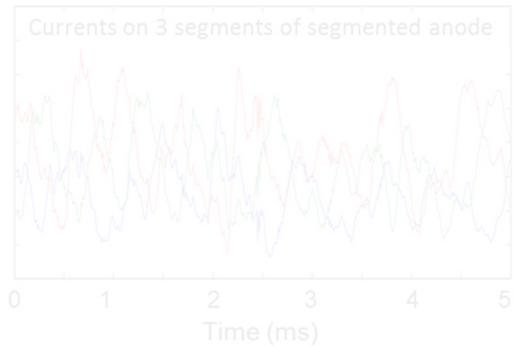
Is there a first-principles explanation for this mode and how it impacts electron transport?



- Energy source from gradients ✓
- Transport due to electric field perturbations in Hall direction ✓

Long-wave length structures may play an important role in cross-field transport in these devices

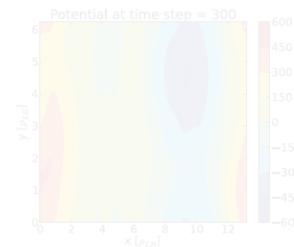
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- Direct measurements of cross-field current from this mode ✓
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On-going area of research

on transport be modeled self-ed transport dominant?



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Case Study 2: Micro-scale instabilities in plasma





Case Study 2: Micro-scale instabilities in plasma

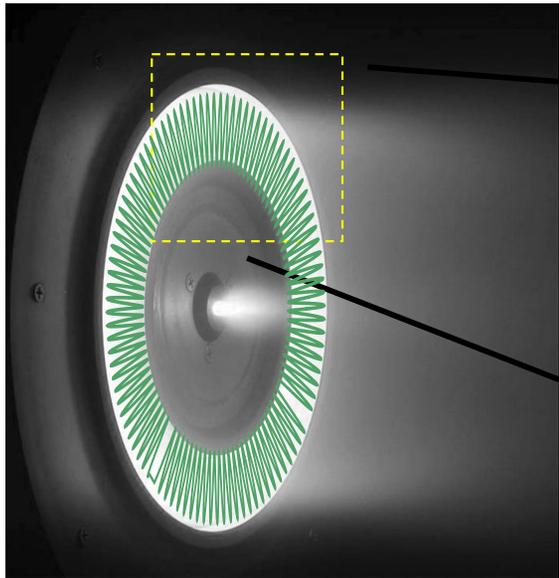
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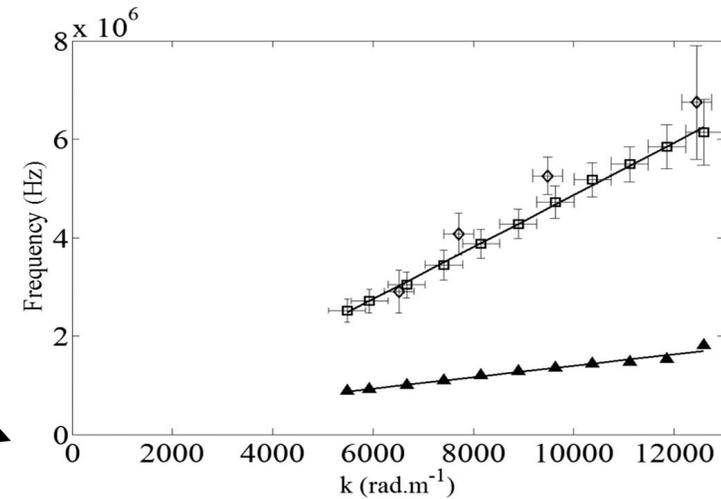


Case Study 2: Micro-scale instabilities in plasma

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Experimental dispersion relation of small-scale oscillations in Hall direction



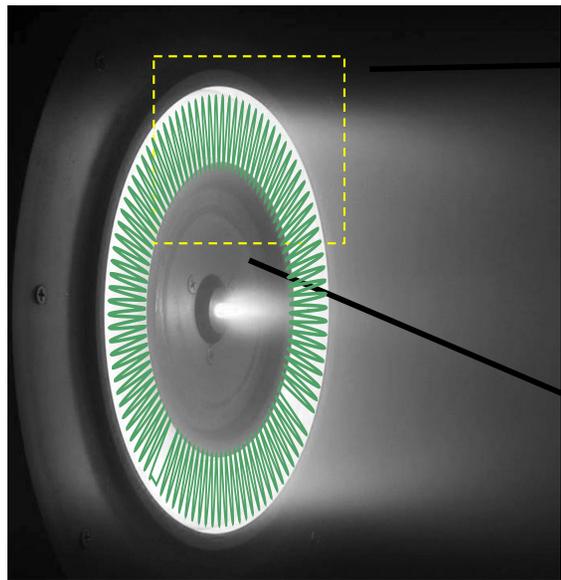
S. Tsikata, N. Lemoine, V. Pisarev, and D. Grésillon, *Physics of Plasmas*. Vol. 16., No. 3. 2009.

- Wavelengths < 1 mm
- Dispersion is acoustic-like
- Modes are incoherent

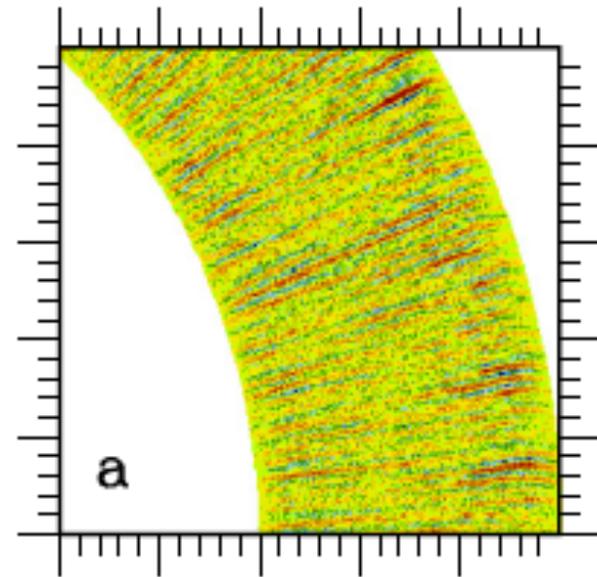


Case Study 2: Micro-scale instabilities in plasma

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Density fluctuations from PIC, 2D model



Sampling of other numerical models showing instability

* A. Héron and J. C. Adam, “Anomalous conductivity in Hall thrusters: Effects of the non-linear coupling of the electron-cyclotron drift instability with secondary electron emission of the walls.” *Physics of Plasmas* 20 , 082313 (2013);

- J. C. Adam , A. Héron , and G. Laval, *Physics of Plasmas* 11 , 295 (2004)
- A. Ducrocq , J. C. Adam , A. Héron , and G. Laval, *Physics of Plasmas* 13 , 102111 (2006);
- J.P. Boeuf. *Frontiers in Physics*, Vol. 2, No. 74, (2014)
- T. Lafleur , , S. D. Baalrud , and P. Chabert, *Physics of Plasmas* 23 , 053502 (2016);
- V Croes et al. *Plasma Sources Sci. Technol.* 26 (2017)



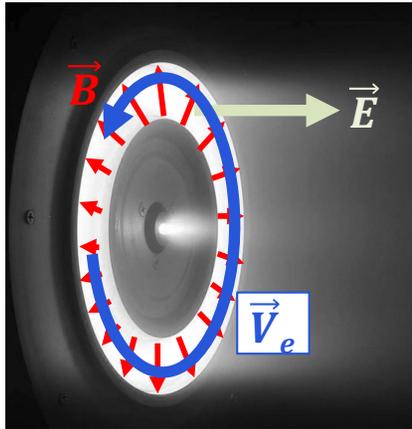
Case Study 2: Micro-scale instabilities in plasma

2. Is there a first-principles explanation for this mode and how it impacts electron transport?

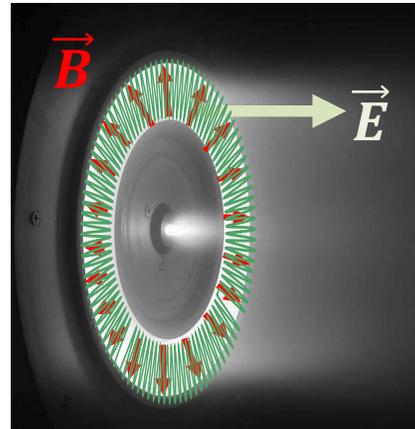


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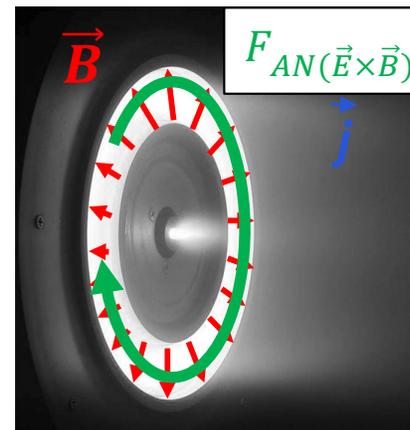


1) Strong $E \times B$ drift between electrons and ions

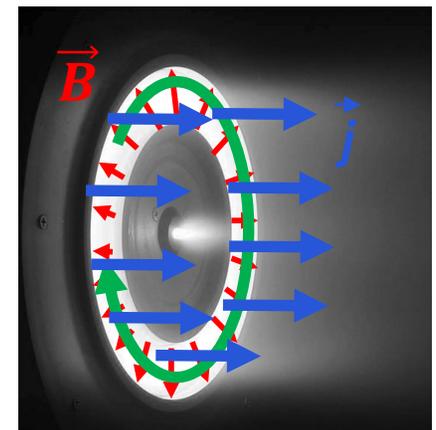


2) Azimuthal electron cyclotron drive instability (ECDI) driven unstable by drift through inverse cyclotron or Landau damping

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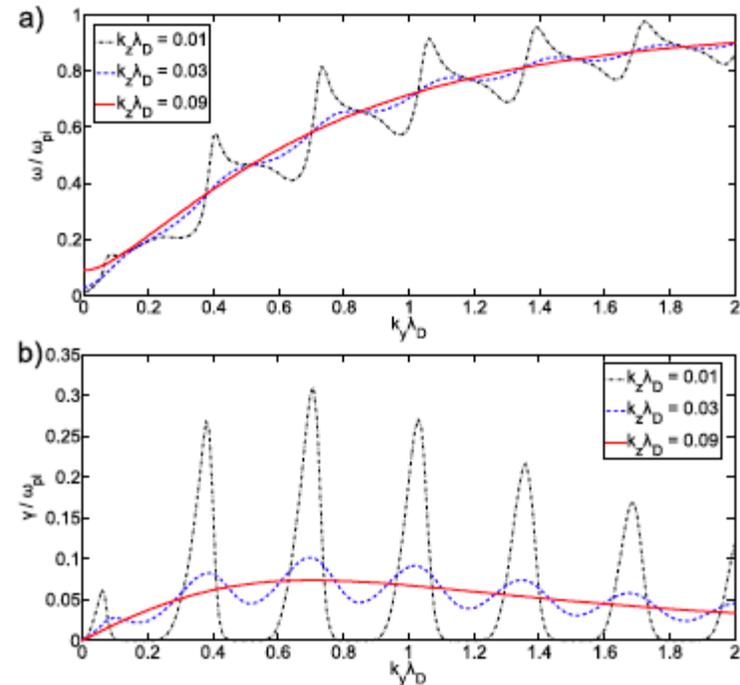
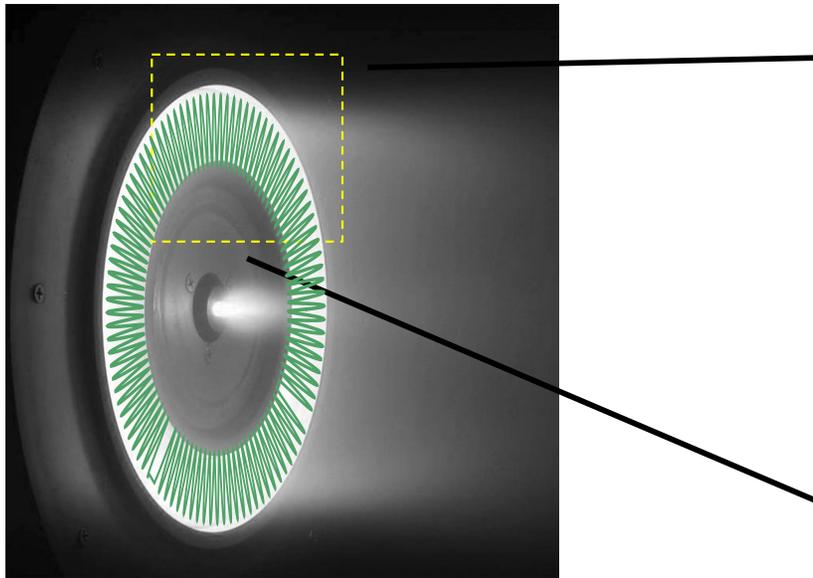


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Positive growth from linear dispersion relation

Linear dispersion relation for electron cyclotron drift instability*



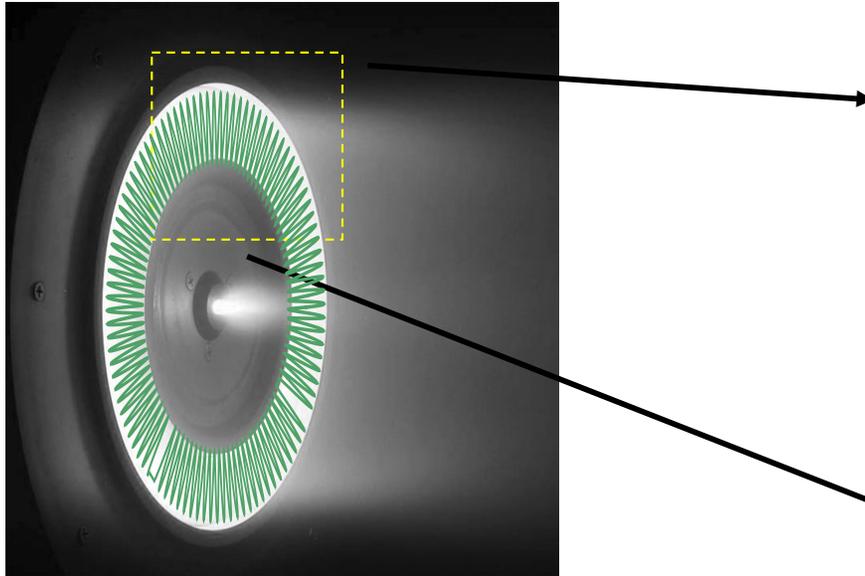
* J. Cavalier et al, "Hall thruster plasma fluctuations identified as the $E \times B$ electron drift instability: Modeling and fitting on experimental data," *Physics of Plasmas* (1994-present) 20 , 082107 (2013);



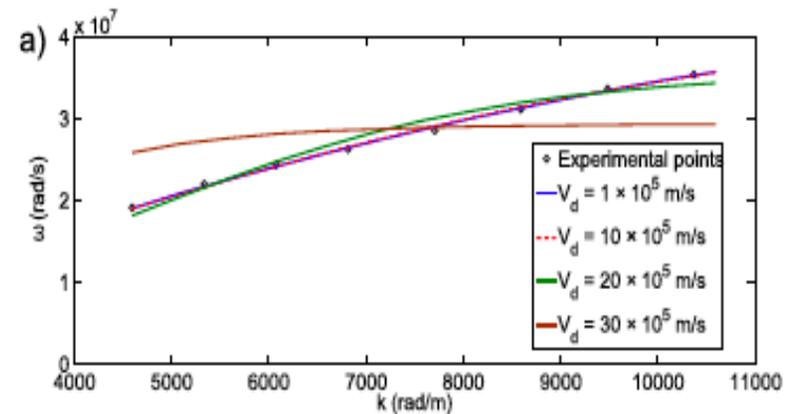
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Linear dispersion vs. experimental data in $E \times B$ direction*



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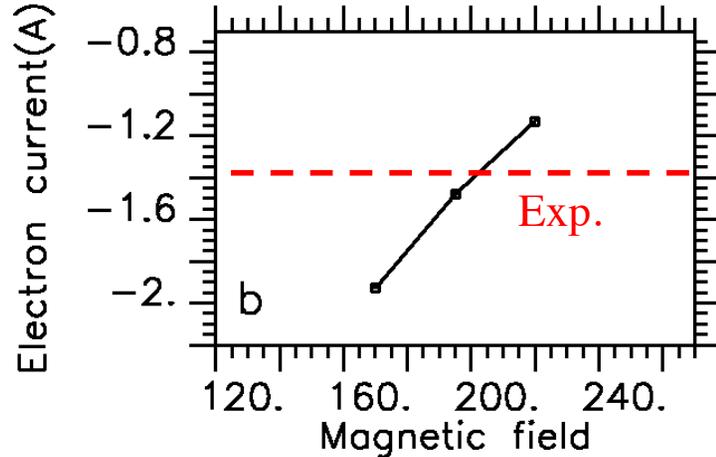
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Case Study 2: Micro-scale instabilities in plasma

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Numerical evidence from Z- θ
PIC code of thruster



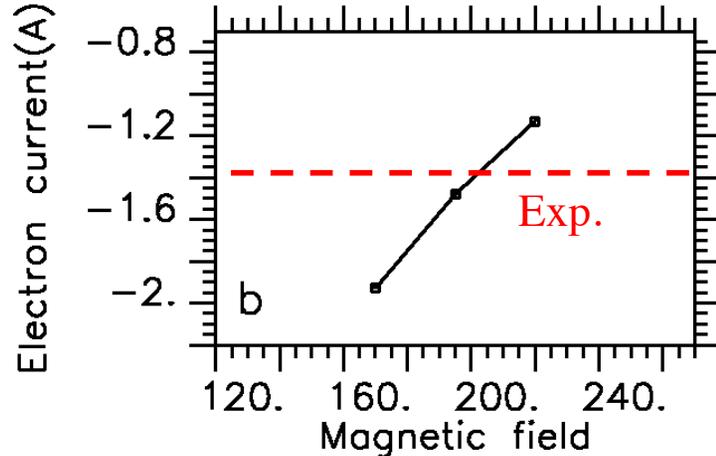
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- Caveat: codes do not capture all experimental properties of observed waves



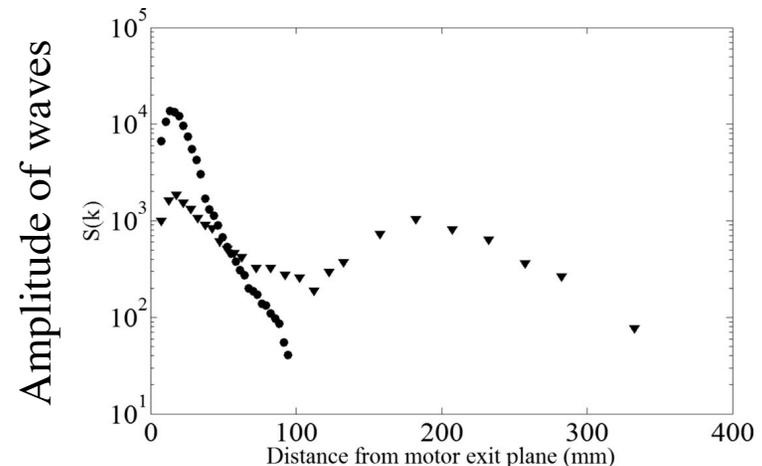
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Experimental evidence



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- PIC numerical codes seem to indicate instability can drive most of transport
- Caveat: codes do not capture all experimental properties of observed waves
- Experimental measurements show spatial dependence of wave energy that is commensurate with electron mobility
- No direct measurements yet of cross-field electron current from these modes.



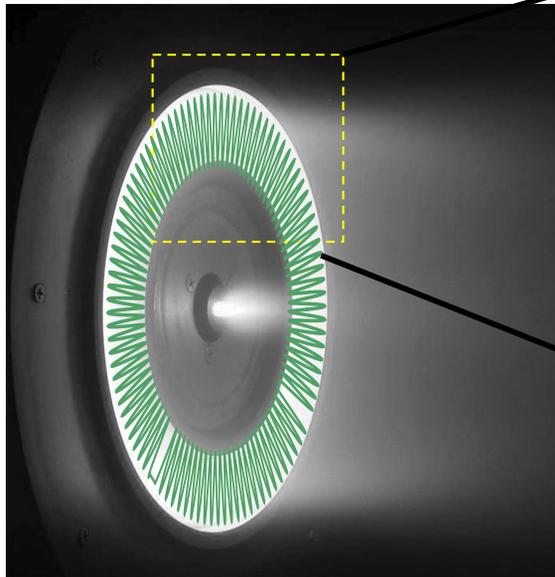
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4. Can the impact of this mode on transport be modeled self-consistently and is the modeled transport dominant?

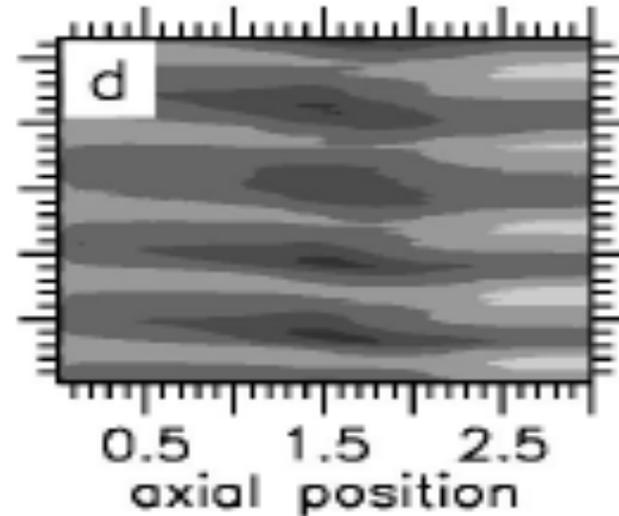


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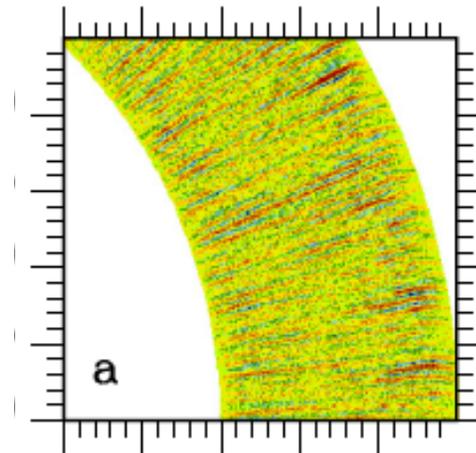


Density from Z- θ PIC simulation



J. C. Adam , A. Héron , and G. Laval, *Physics of Plasmas* 11 , 295 (2004)

Density fluctuations from PIC, 2D model



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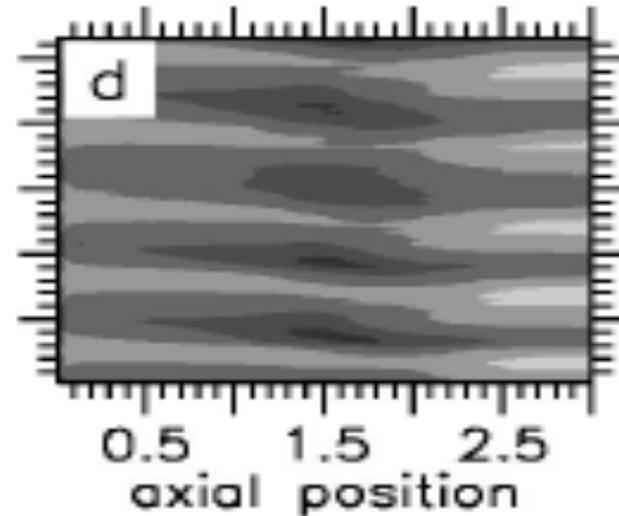


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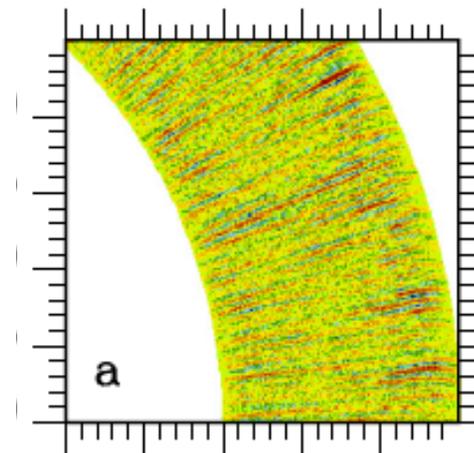
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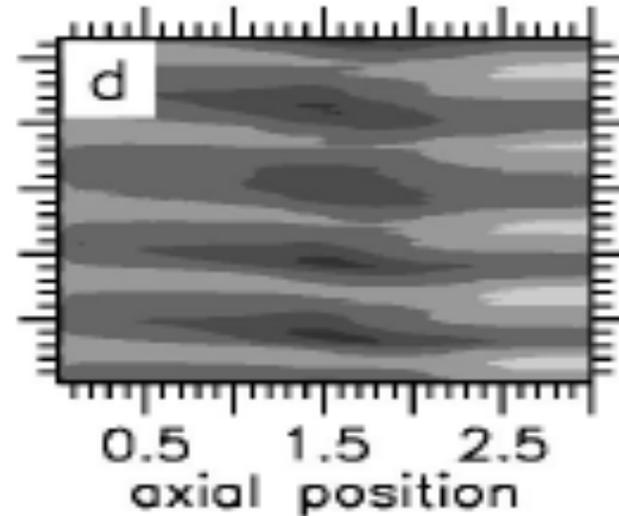


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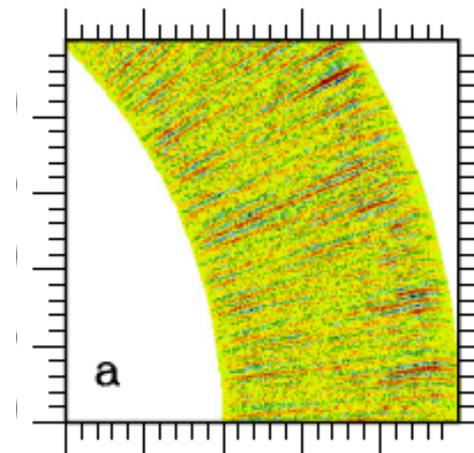
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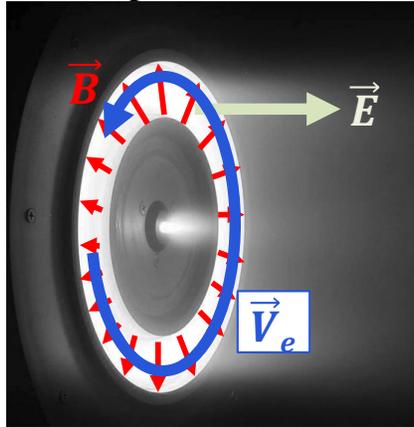
* A. Héron and J. C. Adam, Physics of Plasmas 20 , 082313 (2013);

Alternative approach: Fluid model

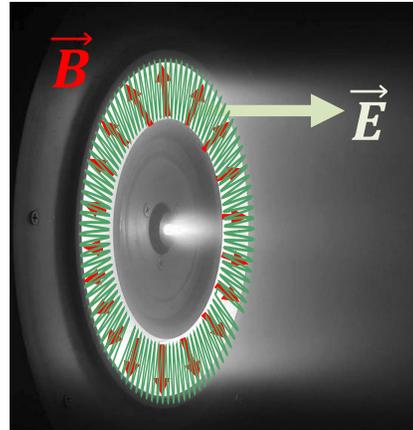


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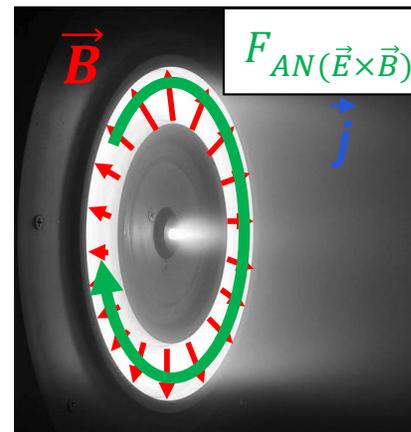
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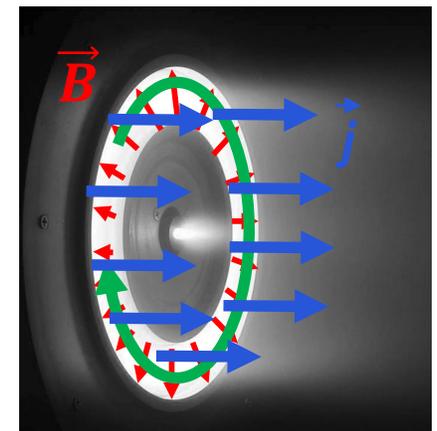
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How do you model a kinetic, small-scale effect like ECDI with a fluid-hierarchy?

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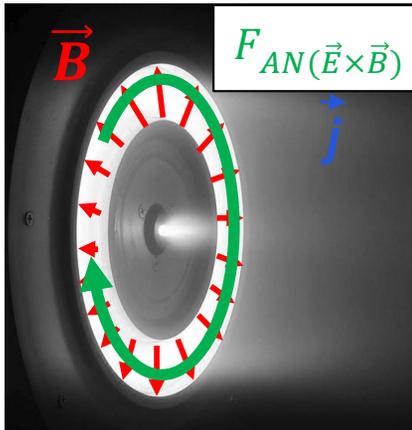


Case Study 2: Micro-scale instabilities in plasma

4. Can the impact of this mode on transport be modeled self-consistently and is the modeled transport dominant?

Fluid approach

How do you model a kinetic, small-scale effect like ECDI with a fluid-hierarchy?



$$j_{\vec{E}} = \frac{1}{(1 + \Omega_e^2)} \left[\frac{E}{\eta} + \Omega_e^2 n \frac{F_{AN(\vec{E} \times \vec{B})}}{B} \right]$$

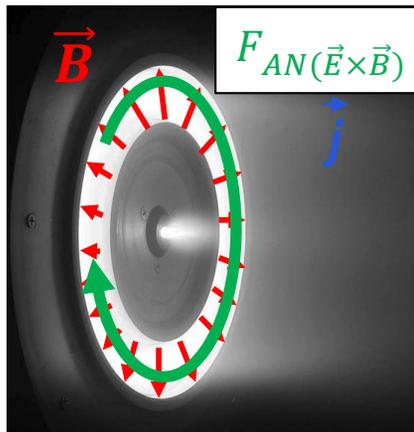


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NUCLEAR FUSION 17 6 (1977)

Review Paper

ANOMALOUS TRANSPORT IN HIGH-TEMPERATURE PLASMAS WITH APPLICATIONS TO SOLENOIDAL FUSION SYSTEMS

R.C. DAVIDSON*
 Department of Physics and Astronomy,
 University of Maryland,
 College Park,
 Maryland

N.A. KRALL
 Laboratory for Applied Plasma Studies,
 Science Applications Inc.,
 La Jolla,
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 United States of America



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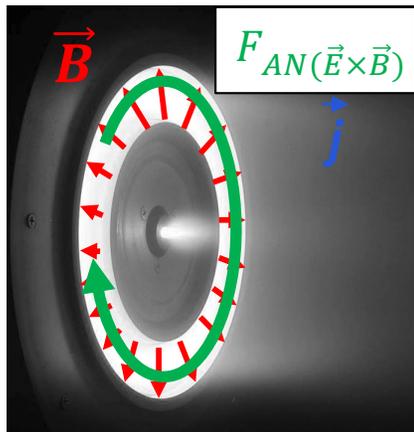
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Quasilinear theory

Saturation closure

Hybrid codes



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 California,
 United States of America



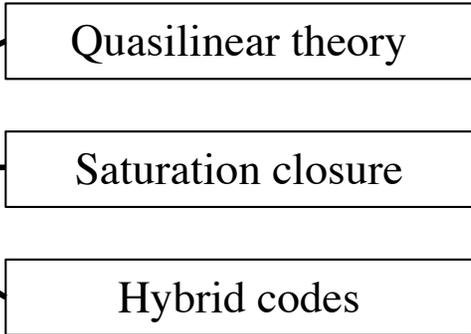
Case Study 2: Micro-scale instabilities in plasma

4. Can the impact of this mode on transport be modeled self-consistently and is the modeled transport dominant?

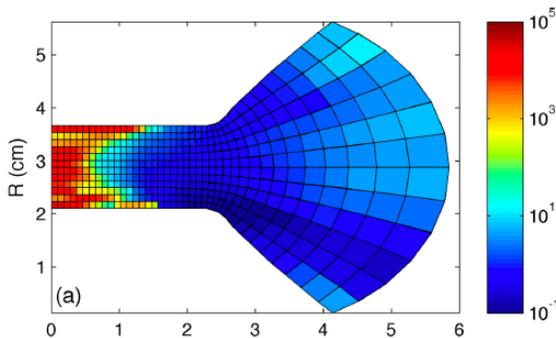
Fluid approach

How do you model a kinetic, small-scale effect like ECDI with a fluid-hierarchy?

$$j_{\vec{E}} = \frac{1}{(1 + \Omega_e^2)} \left[\frac{E}{\eta} + \Omega_e^2 n \frac{F_{AN(\vec{E} \times \vec{B})}}{B} \right]$$



Entropy argument for closure



M. Cappelli et al. Physics of Plasmas 22 , 114505 (2015)

Full QL theory

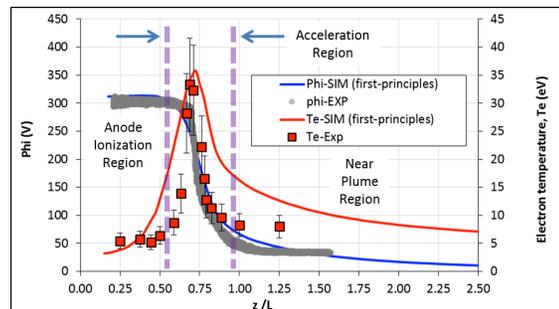
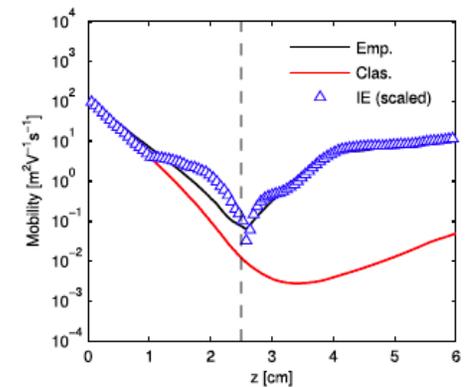


Figure 7. Potentials and electron temperatures from the self-consistent steady-state solution along the channel centerline compared with laboratory measurements on the H6US thruster

I. Katz, I. Mikellides, B. Jorns, and A. Lopez-Ortega. 34th IEPC. Kobe, Japan. IEPC-2015-402.

QL theory with saturation



T. Lafleur , , S. D. Baalrud , and P. Chabert. Physics of Plasmas 23 , 053503 (2016);



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Fluid approach

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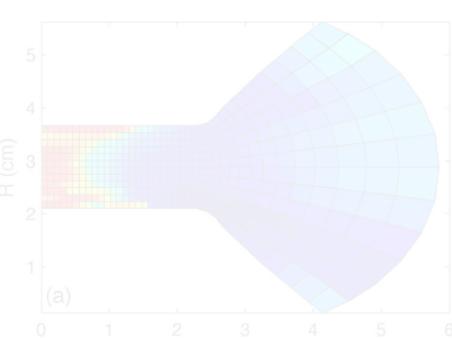
No closure attempt has self-consistently captured the cross-field transport yet

Quasilinear theory

Saturation closure

Hybrid codes

There are many open questions about the validity of using fluid approximations and closures to model kinetic effects



- Growth rates
- Energy transfer across length scales
- Saturation

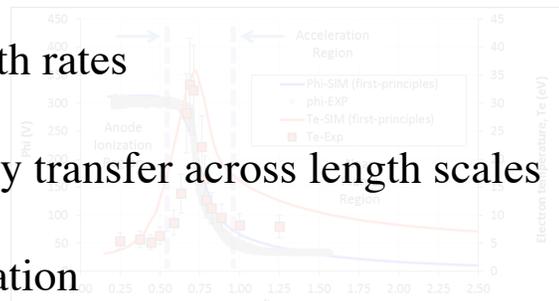
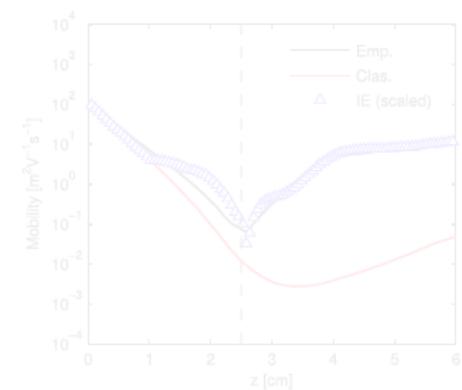


Figure 7. Potentials and electron temperatures from the self-consistent steady-state solution along the channel centerline compared with laboratory measurements on the HGLS thruster



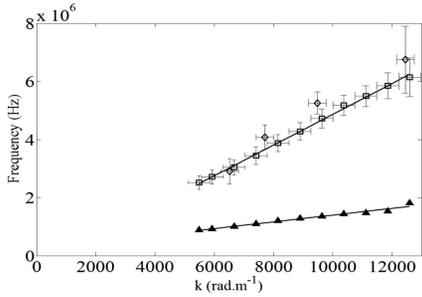
M. Cappelli et al. Physics of Plasmas 22 , 114501 (2015) • 2015 Non-thermalized plasmas and A. Lopez-Ortega. 34th IEPC, Kobe, Japan. IEPC-2015-402.

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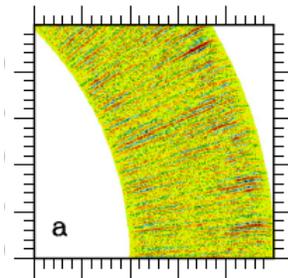


Case Study 2: Micro-scale instabilities in plasma

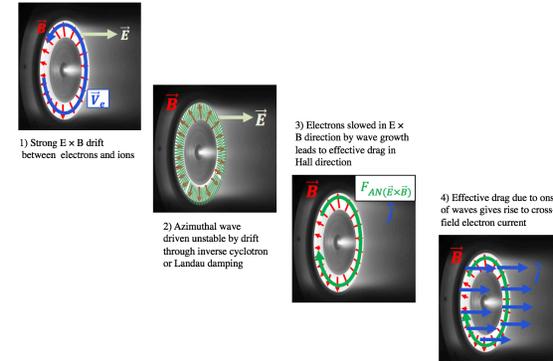
Is there evidence that this mode should exist in the system?



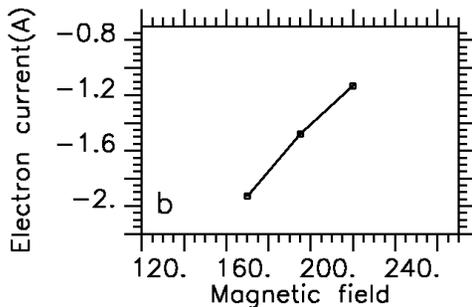
- Experimental ✓
- Numerical ✓
- Analytical ✓



Is there a first-principles explanation for this mode and how it impacts electron transport?



Is there evidence that this mode is sufficiently strong to drive observed cross-field electron transport?



- Numerical simulations suggest that it can be a dominant mode ✓
- Limited direct experimental evidence of electron transport (?)

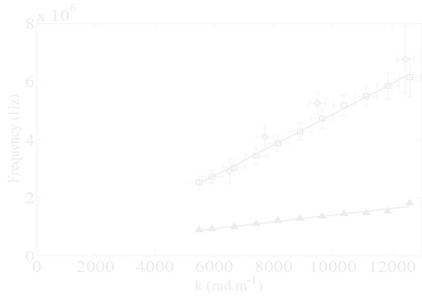
Can the impact of this mode on transport be modeled self-consistently and is the modeled transport dominant?

- Multiple hierarchies being explored that predict some level of transport ✓
- PIC models do not have the computational power (?)
- Open question as to how to approximate effect in fluid models (?)



Case Study 2: Micro-scale instabilities in plasma

Is there evidence that this mode should exist in the system?



- Experimental ✓
- Numerical ✓
- Analytical ✓

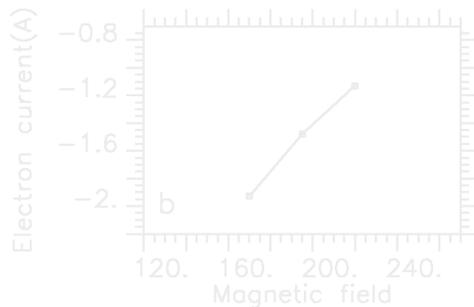
Long-wave length structures may play an important role in cross-field transport in these devices

Is there a physical explanation for how this mode grows and impacts electron transport?



On-going area of research

Is there evidence that this mode drive observed cross-field electron transport?



- Numerical simulations suggest that it can be a dominate mode ✓
- Limited direct experimental evidence of electron transport (?)

Can cross-field transport be modeled self-consistently and is the modeled transport dominant?

- Multiple hierarchies being explored that predict some level of transport ✓
- PIC models do not have the computational power (?)
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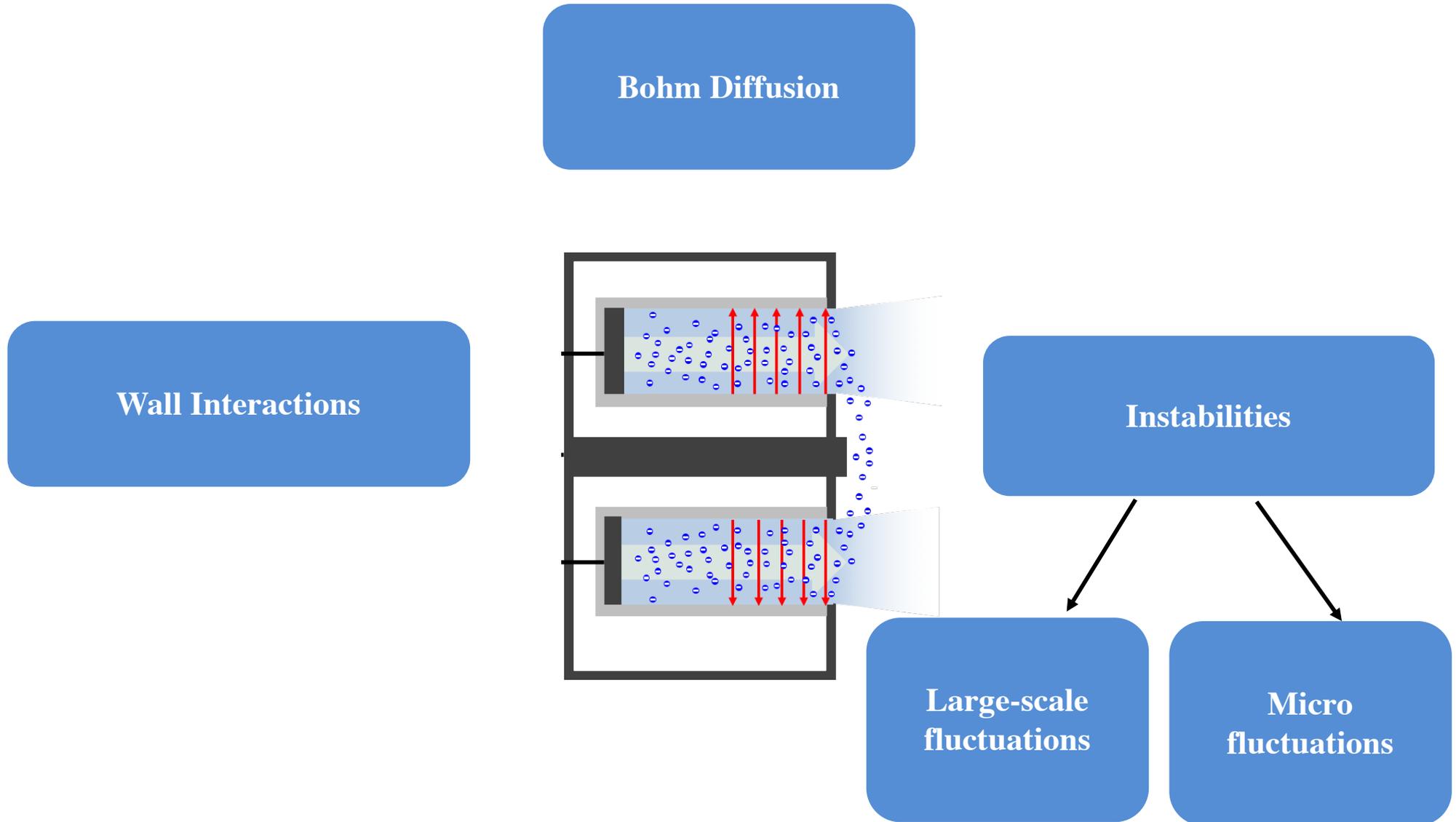


Recap

- There is probably some anomalous force in the Hall direction driving enhanced cross-field transport
- Plasma instabilities with components in the Hall the direction can contribute to this anomalous force
- There are many known instabilities in Hall thrusters. It is important to have criteria to differentiate and evaluate their importance for contributing to the cross-field transport
- Measured against these criteria, two promising and popular candidates are **large scale, near-coherent** fluctuations and **microturbulence**.

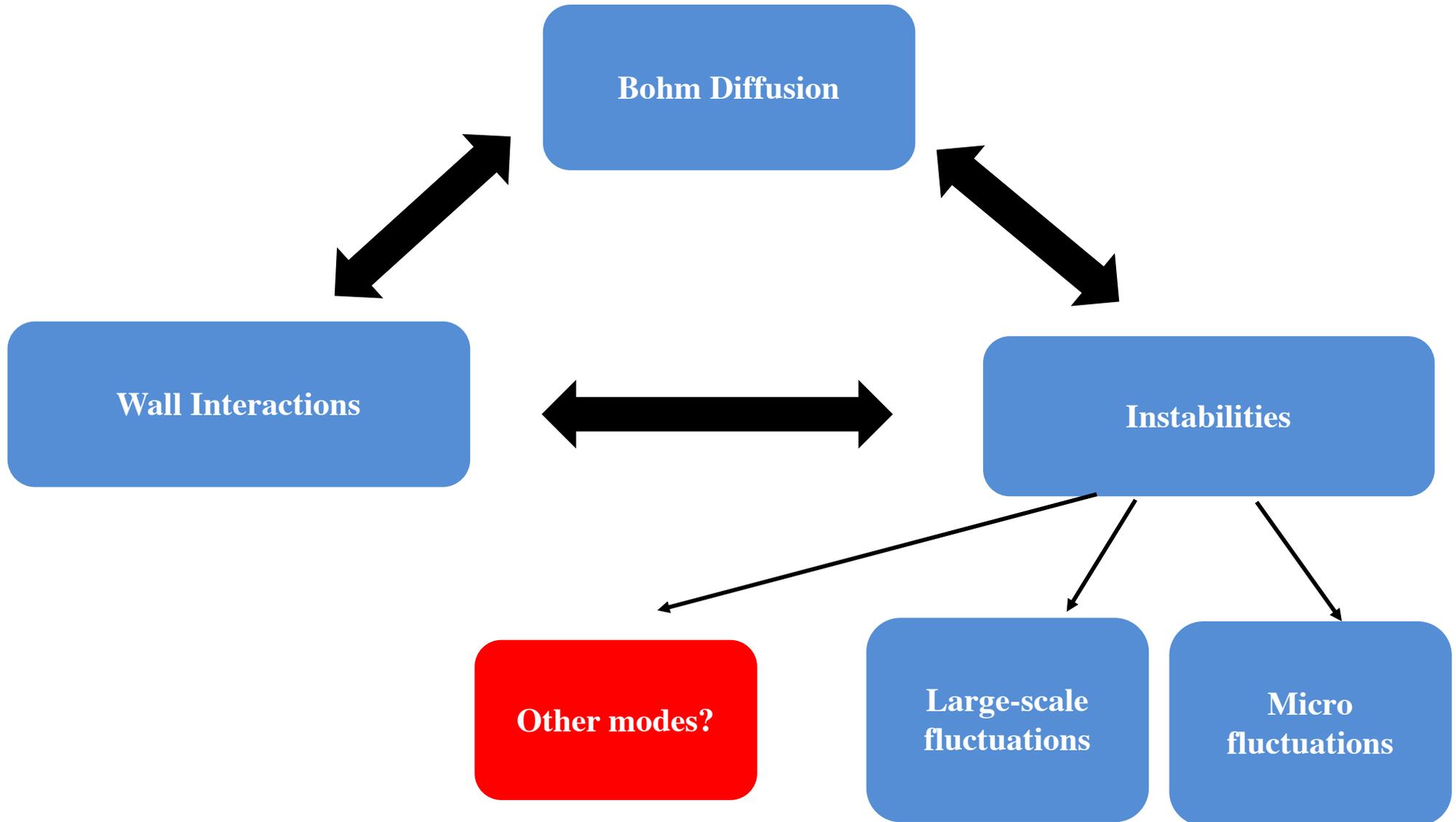


The actual physical system is more complex





The actual physical system is more complex





Moving Forward

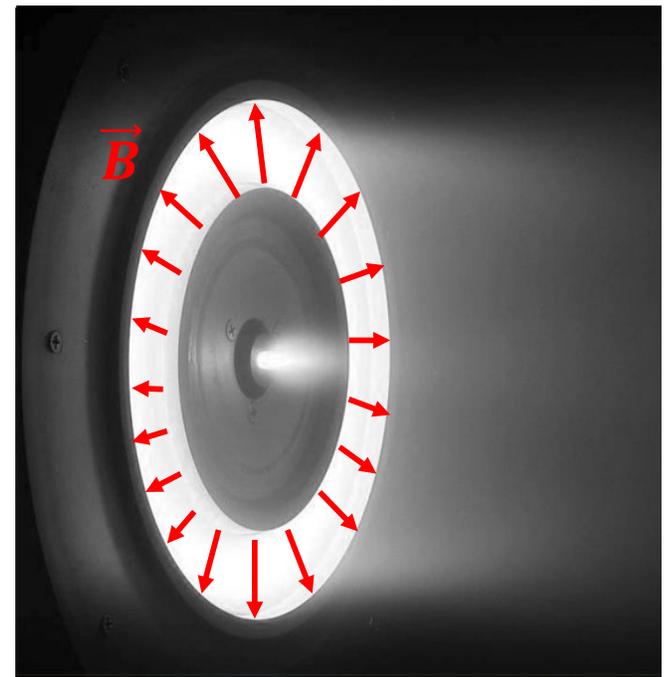
- **Need to continue to explore first-principles formulations and numerical efforts in parallel**
 - Computational speed and fidelity of PIC
 - Incorporating (where appropriate) kinetic effects into fluid codes
 - Understanding how disparate effects interact and convolve
- **Need to develop experimental tools and diagnostics to better characterize instabilities and their impact on transport**
- **Need to continue to develop tools to identify instabilities in the thruster that may contribute to transport (next talk)**



Need for additional forcing terms to drive cross-field transport

Classical (collisional) Ohm's Law

$$\eta \vec{j} = \vec{E} - \frac{1}{qn} \vec{j} \times \vec{B} \quad \eta = \text{resistivity}$$





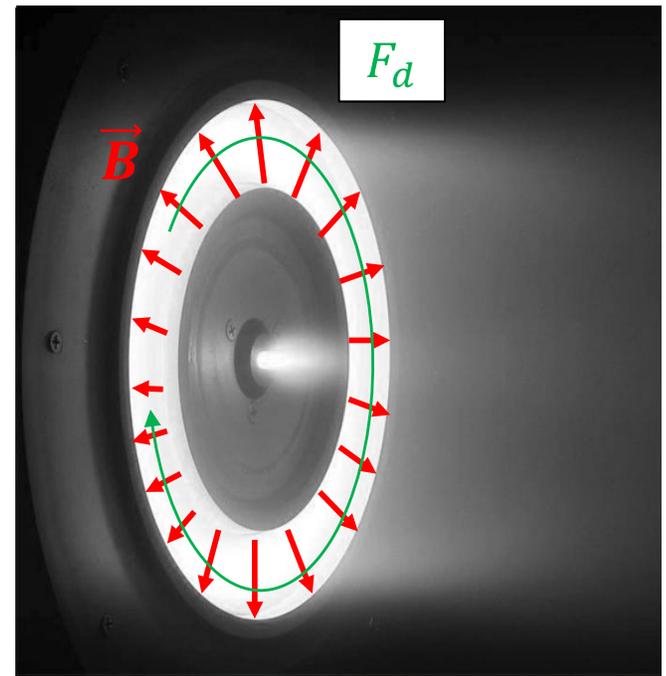
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Resistive drag term " \vec{F}_d " against $\mathbf{E} \times \mathbf{B}$ drift allows electrons to cross field lines





Need for additional forcing terms to drive cross-field transport

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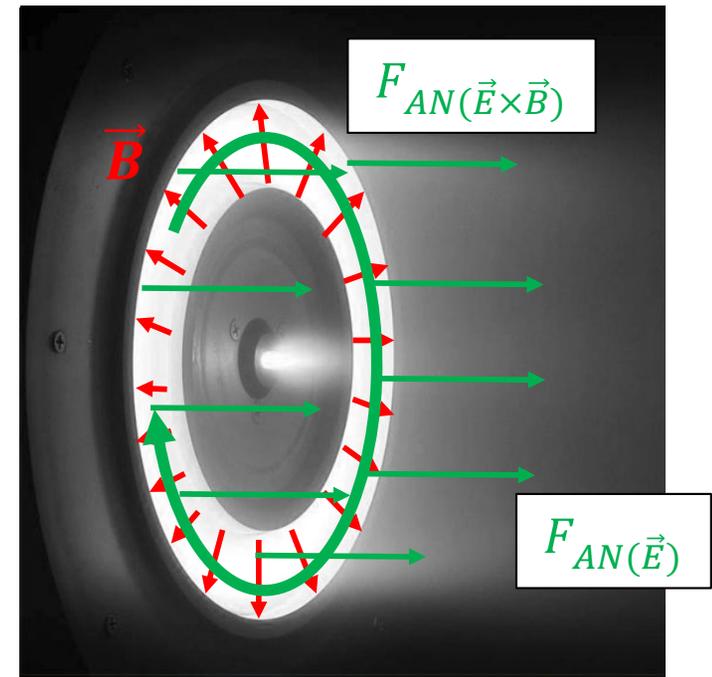
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What if there are other forcing terms?

$$-\frac{1}{q} \vec{F}_{AN} + \eta \vec{j} = \vec{E} - \frac{1}{qn} \vec{j} \times \vec{B}$$





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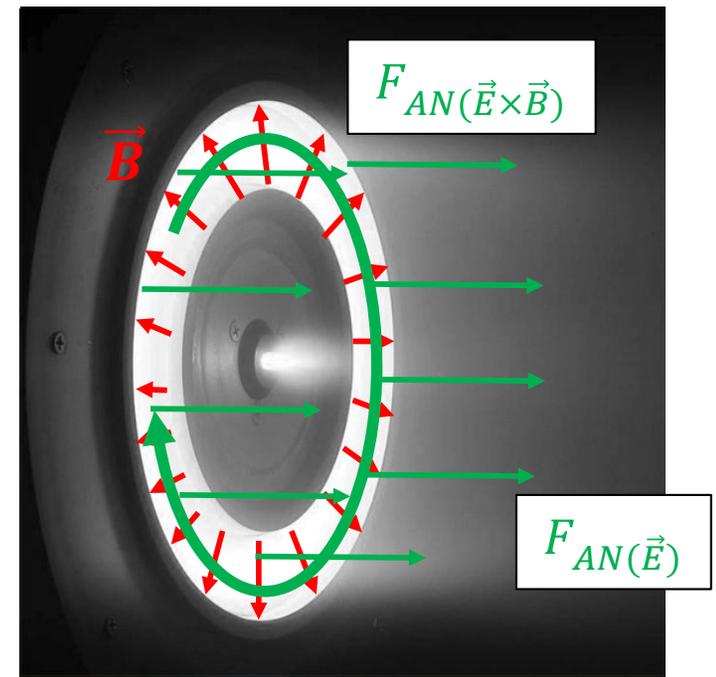
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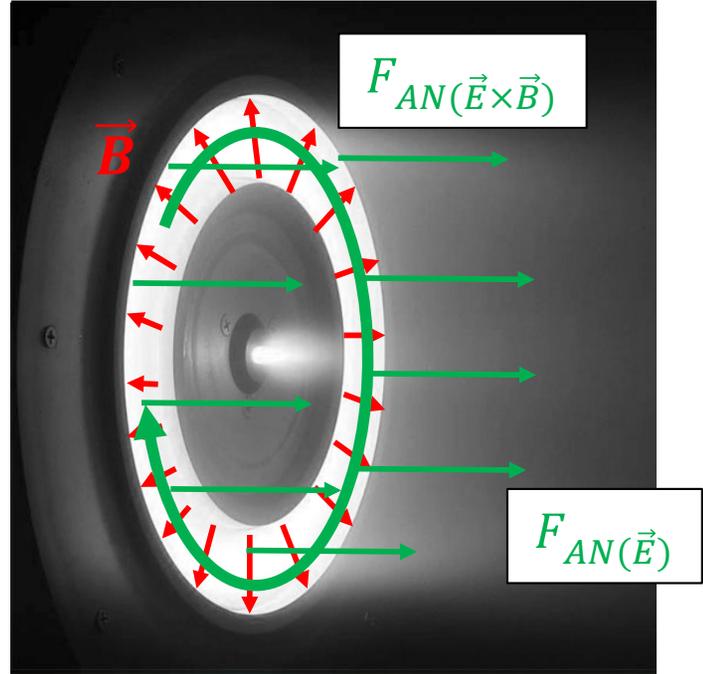
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Classical

Anomalous



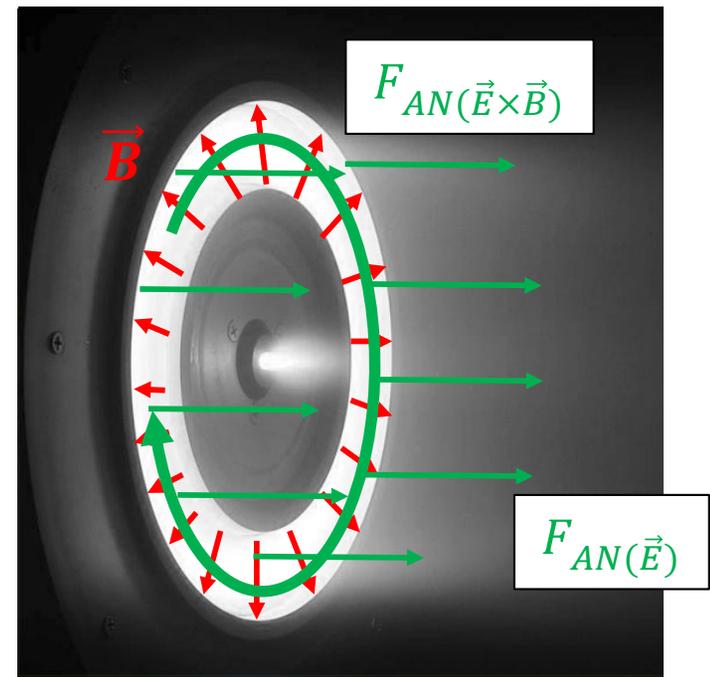
Additional force term can provide cross-field current



Cross-field Transport in Partially Magnetized Plasmas

Electric field is very strong in Hall accelerator
and classical Hall parameter is high ($\Omega_e \gg 1$)

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Cross-field Transport in Partially Magnetized Plasmas

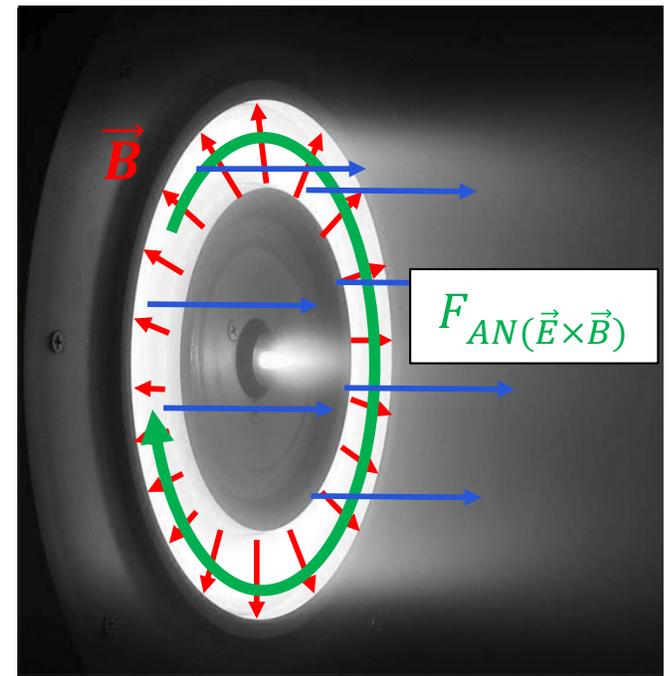
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$$j_{\vec{E}(AN)} = n \frac{F_{AN(\vec{E} \times \vec{B})}}{B}$$

Cross-field current largely driven by
force acting in the Hall direction





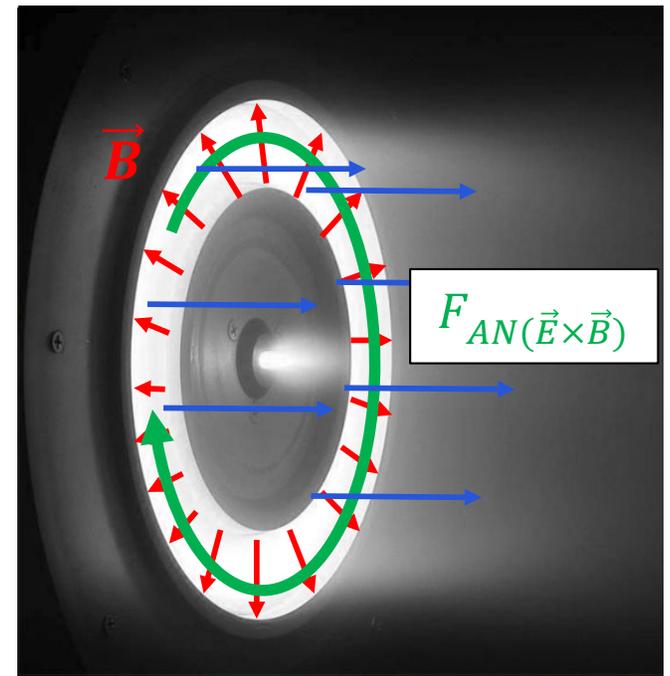
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Cross-field current largely driven by
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What can provide this additional force?