Electron drift instabilities in ExB plasmas: kinetic theory and PIC simulations

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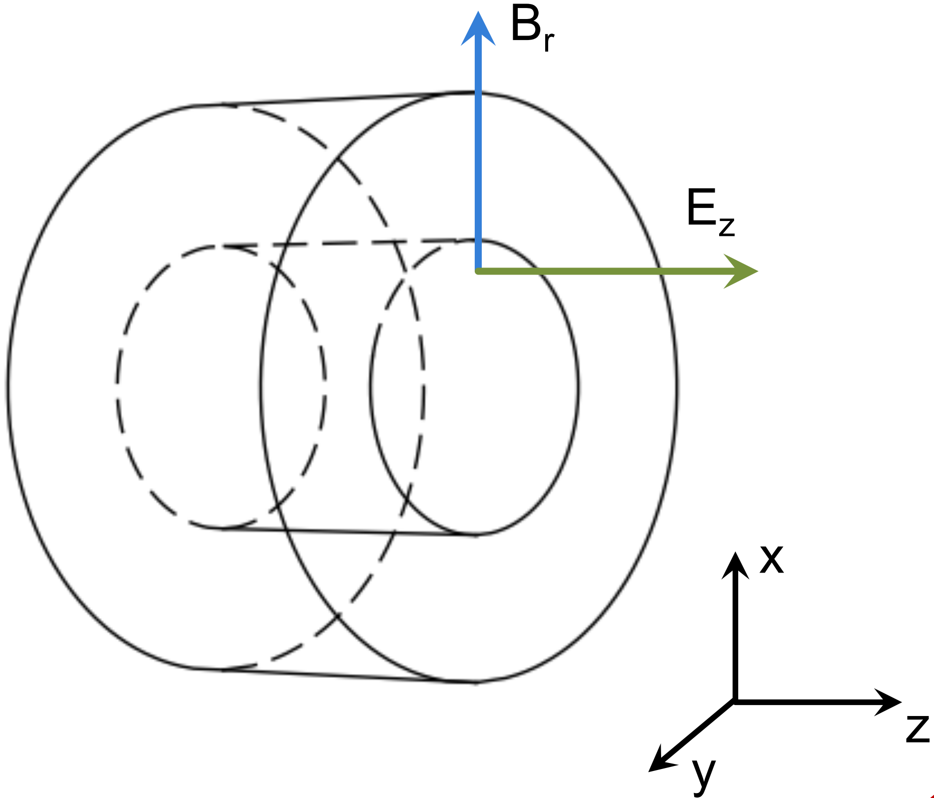
A. Kinetic theory

3D Dispersion relation

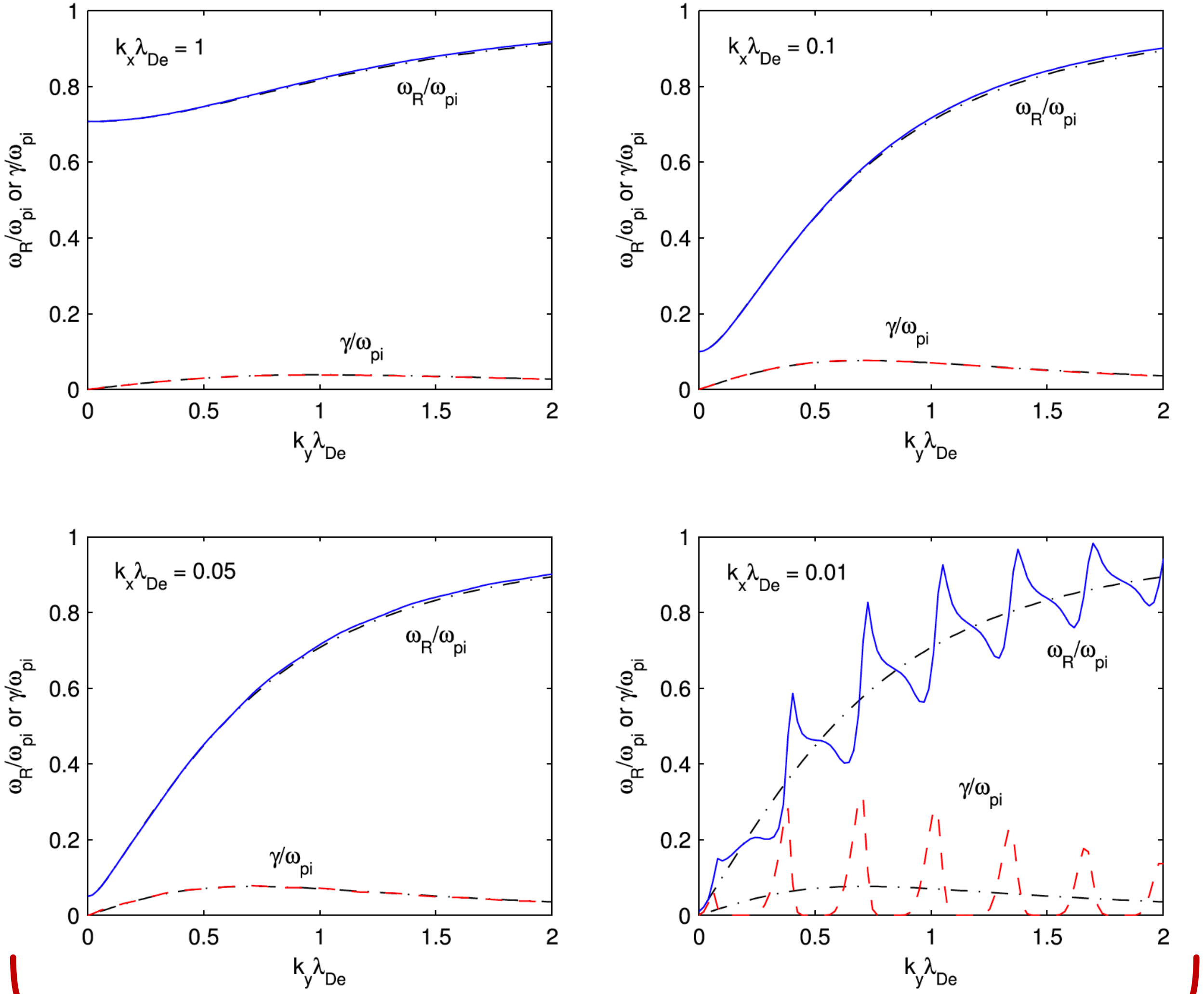


$$\beta = \frac{k_\perp^2 v_{Te}^2}{2\omega_{ce}^2}$$

$$0 = 1 - \frac{\omega_{pi}^2}{k^2 v_{Ti}^2} Z' \left(\frac{\omega - \mathbf{k} \cdot \mathbf{v}_i}{k v_{Ti}} \right) + \frac{1}{k^2 \lambda_{De}^2} \left[1 + \left(\frac{\omega - k_y v_{ey}}{k_x v_{Te}} \right) \sum_{n=-\infty}^{\infty} e^{-\beta} I_n(\beta) Z \left(\frac{\omega - k_y v_{ey} - n\omega_{ce}}{k_x v_{Te}} \right) \right]$$

A. Ducrocq *et al*, *Phys. Plasmas* **13**, 102111 (2006); T. Lafleur *et al*, *Phys. Plasmas* **23**, 053503 (2016)

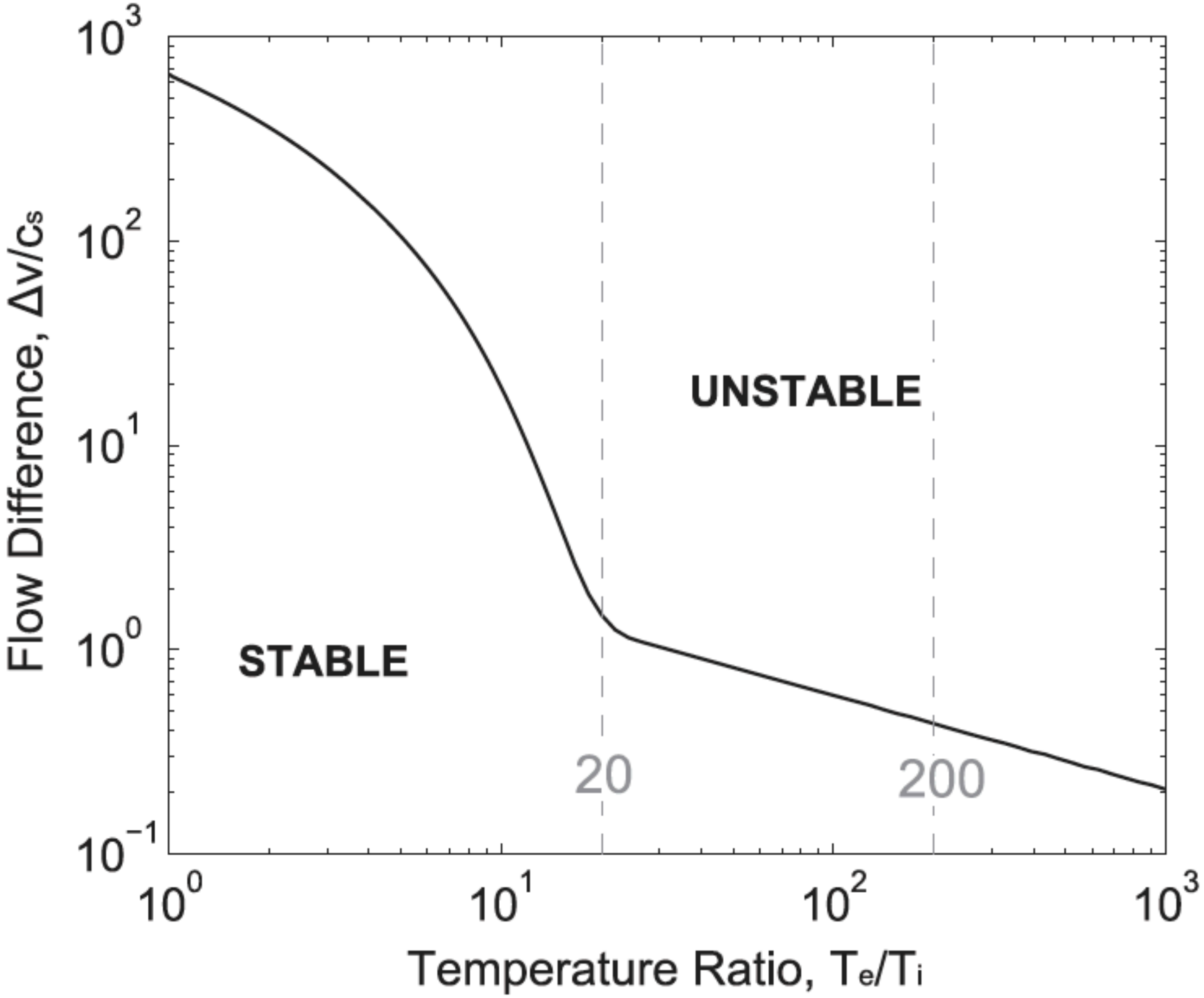
Electron drift instability (EDI)



$$0 = 1 - \frac{\omega_{pi}^2}{k^2 v_{Ti}^2} Z'\left(\frac{\omega - \mathbf{k}\cdot\mathbf{v}_i}{k v_{Ti}}\right) - \frac{\omega_{pe}^2}{k^2 v_{Te}^2} Z'\left(\frac{\omega - \mathbf{k}\cdot\mathbf{v}_e}{k v_{Te}}\right)$$

J. Cavalier *et al*, *Phys. Plasmas* **20**, 082107 (2013); T. Lafleur *et al*, *Phys. Plasmas* **23**, 053503 (2016)

Instability thresholds



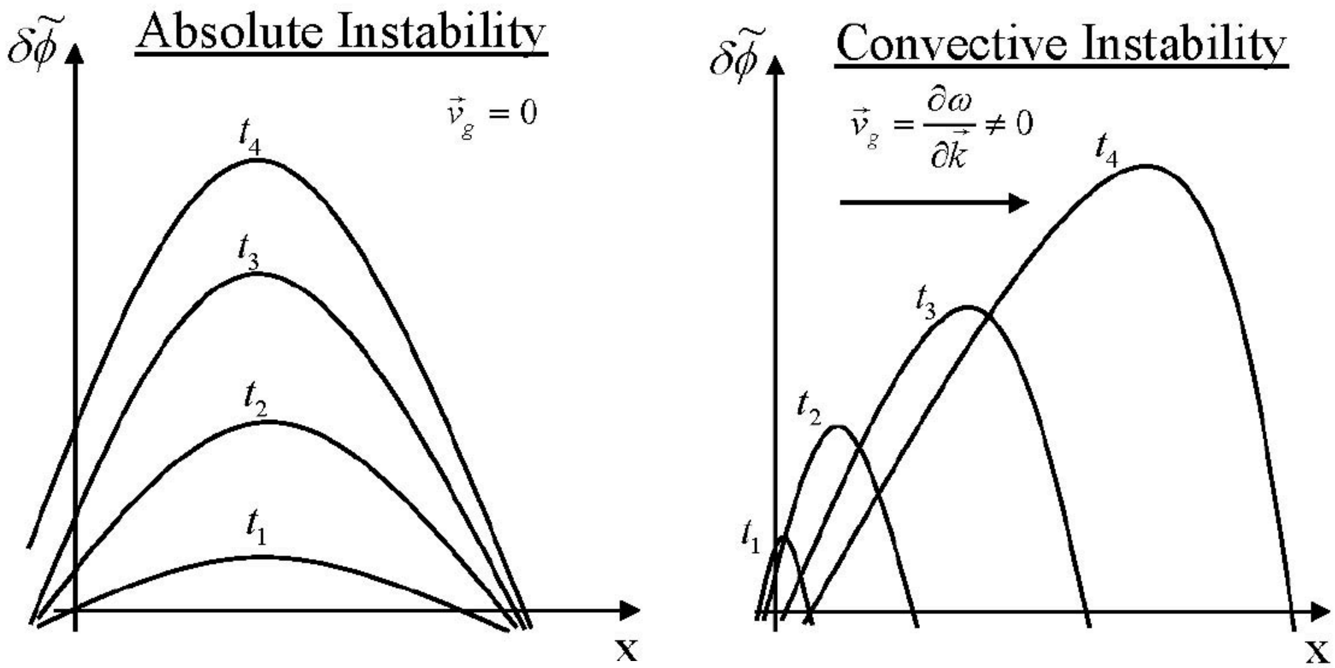
Two EDIs:

1. Azimuthal
2. Axial

$$\frac{kc_s}{\sqrt{1+k^2\lambda_{De}^2}}\left[1+\sqrt{\frac{M}{m}}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{2T_i}\frac{1}{1+k^2\lambda_{De}^2}\right)\right] \lesssim |\mathbf{k}\cdot(\mathbf{v}_i-\mathbf{v}_e)|$$

T. Lafleur *et al*, *Plasma Sources Sci. Technol*. **26**, 024008 (2017)

Absolute vs convective instabilities



$$\frac{\partial W}{\partial t} + \nabla \cdot (\mathbf{v}_g W) = 2\gamma W$$

$$\alpha = \frac{\text{convection rate}}{\text{growth rate}} \sim \frac{\omega_{ce}}{\omega_{pe}}$$

T.H. Stix, *Waves in Plasmas* (Springer-Verlag, 1992); T. Lafleur *et al*, *Plasma Sources Sci. Technol.* **26**, 024008 (2017)

Electron-ion friction force

- Split *f* into equilibrium and perturbed parts. Equilibrium kinetic equation:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = -\frac{q_s}{m_s}\left\langle (\delta\mathbf{E} + \mathbf{v} \times \delta\mathbf{B}) \cdot \frac{\partial \delta f_s}{\partial \mathbf{v}} \right\rangle$$

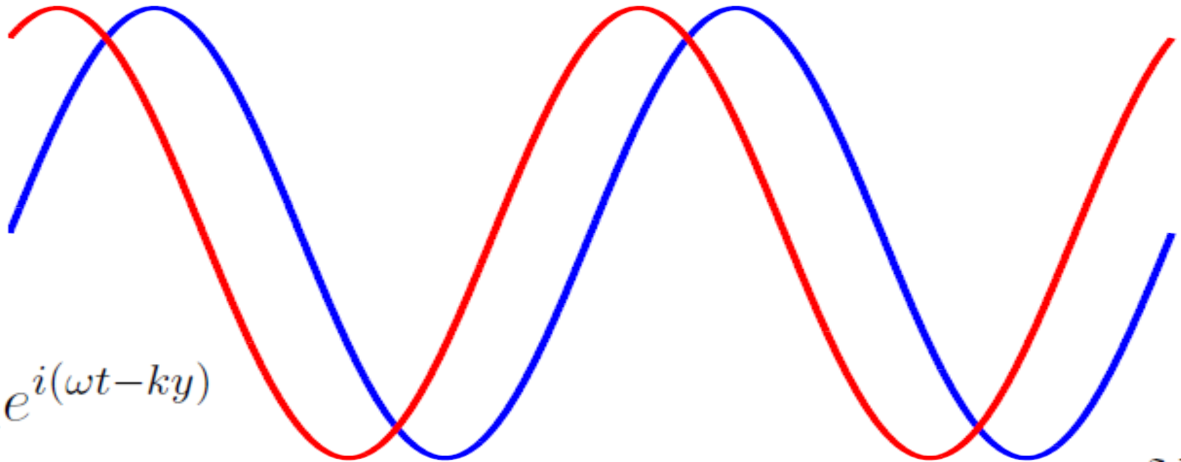
Multiply by *m***v** and integrate over **v**

$$\frac{\partial}{\partial t}(mn_e\mathbf{v}_{de}) + \nabla \cdot (mn_e\mathbf{v}_{de}\mathbf{v}_{de}) = qn_e(\mathbf{E} + \mathbf{v}_{de} \times \mathbf{B}) - \nabla \cdot \mathbf{\Pi}_e + \mathbf{R}_{ei}$$

Electron-ion frictional drag force: $\mathbf{R}_{ei} = q\langle \delta n_e \delta\mathbf{E} \rangle$

Instability: physical picture

- Instability leads to oscillations in the electron density and azimuthal electric field that are **correlated**:



$$\delta n_e = \delta \tilde{n}_e e^{i(\omega t - ky)}$$

$$\delta E = \delta \tilde{E} e^{i(\omega t - ky)}$$

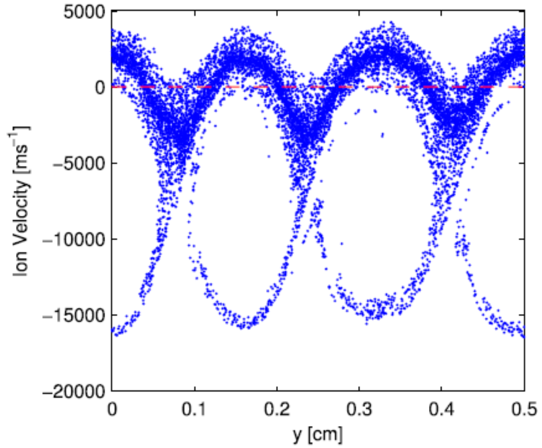
- Averaging over time and space leads to a non-zero friction force:

$$\mathbf{R}_{ei} = q\langle \delta n_e \delta \mathbf{E} \rangle = \frac{q}{2} \mathrm{Re} \left\{ \delta \tilde{n}_e \delta \tilde{E}_\theta^* \right\}$$

- Viewed in this way there are 5 unknowns: ω, k, $|\delta n_e|$, $|\delta E|$, phase shift ($\Delta\theta$)

Finding unknown parameters

- ω,*k* → Dispersion relation
- $|\delta n_e|$ → Vlasov equation
- $|\delta E|$ → Ion trapping



- Phase shift ($\Delta\theta$)

$$\frac{\partial W}{\partial t} + \nabla \cdot (\mathbf{v}_g W) = 2\gamma W \rightarrow \Delta\theta \propto \left.\frac{df_e}{dv}\right|_{v_{res}} \propto \gamma$$

T. Lafleur *et al*, *Phys. Plasmas* **23**, 053503 (2016)

Instability-enhanced friction force

- Solving for the unknowns, we obtain an expression for the instability-enhanced (IE) electron-ion friction:

$$R_{ei}^{IE} \approx \frac{|q|}{4\sqrt{6}} \frac{1}{c_s} \left| \nabla \cdot (\mathbf{v}_i n_e T_e) \right|$$

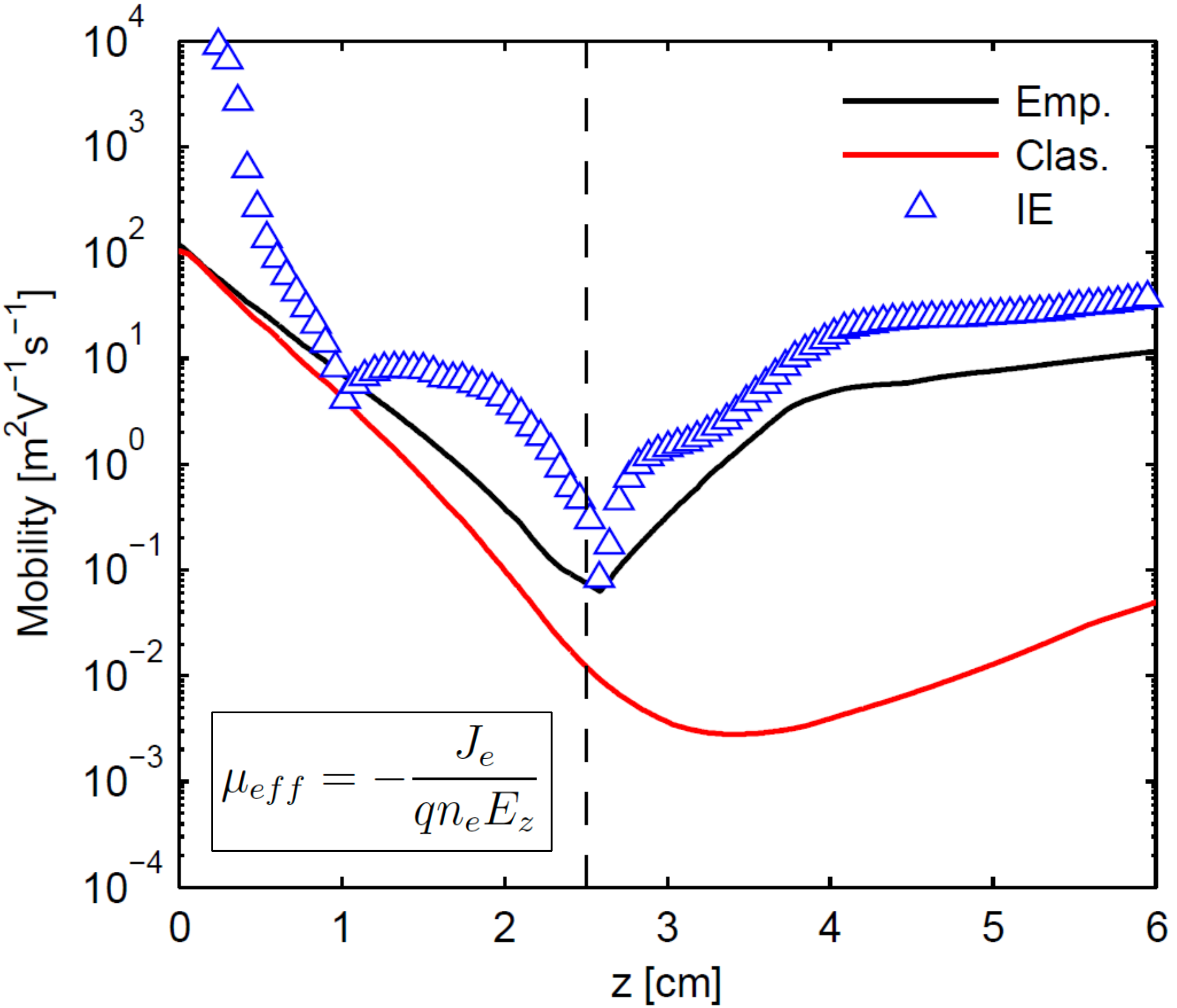
- By comparison, standard electron-ion Coulomb collisions give:

$$R_{ei}^{LB} \approx \frac{16\sqrt{\pi} q^4 n_e^2 v_{ey} \ln \Lambda}{3 \left(4\pi\epsilon_0\right)^2 m v_{Te}^3}$$

- For typical Hall thruster conditions R^{IE}_{ei} is 2-3 orders of magnitude larger than R^{LB}_{ei} and also typical electron-neutral collisional friction

- The instability-enhanced friction can explain anomalous electron mobility/transport observed in experiments

Comparison with exp./sim.



J.C. Adam et al, Plasma Phys. Controlled Fusion **50**, 124041 (2008); T. Lafleur *et al*, *Phys. Plasmas* **23**, 053503 (2016)

Ion rotation and heating

- If instability leads to an electron-ion friction force, then there should also be an equal and opposite ion-electron friction force (i.e. $\mathbf{R}_{ei} = -\mathbf{R}_{ie}$)

- Combining the ion conservation equations with the approximate expression for R_{ei} leads to a prediction of ion rotation in the azimuthal direction, and ion heating:

$$v_{iy} \approx \frac{c_s}{4\sqrt{6}}$$

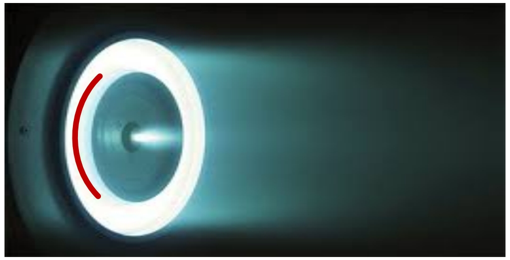
$$T_i \approx \frac{T_e}{30}$$

T. Lafleur *et al*, *Plasma Sources Sci. Technol*. **26**, 024008 (2017)

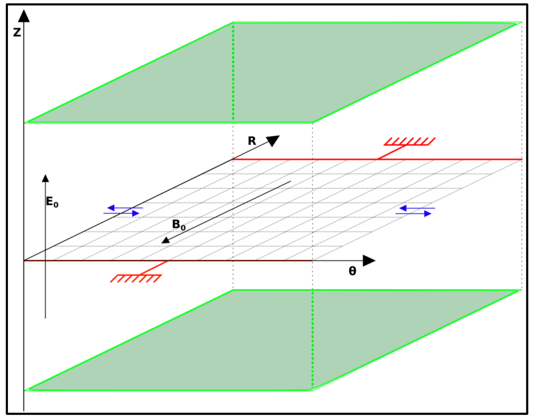
B. PIC simulations

PIC models

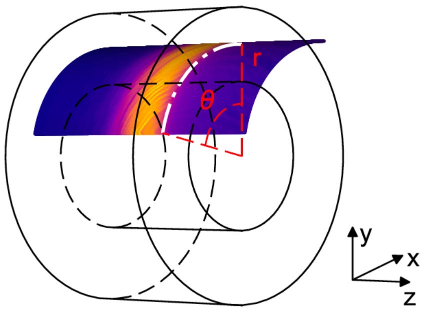
1D "θ"*

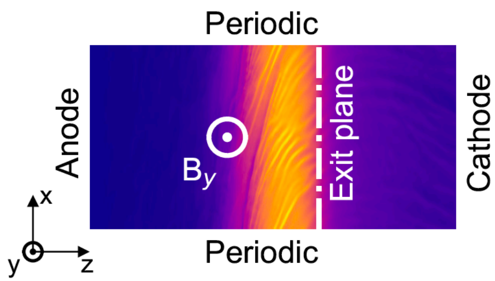


2D "r-θ"



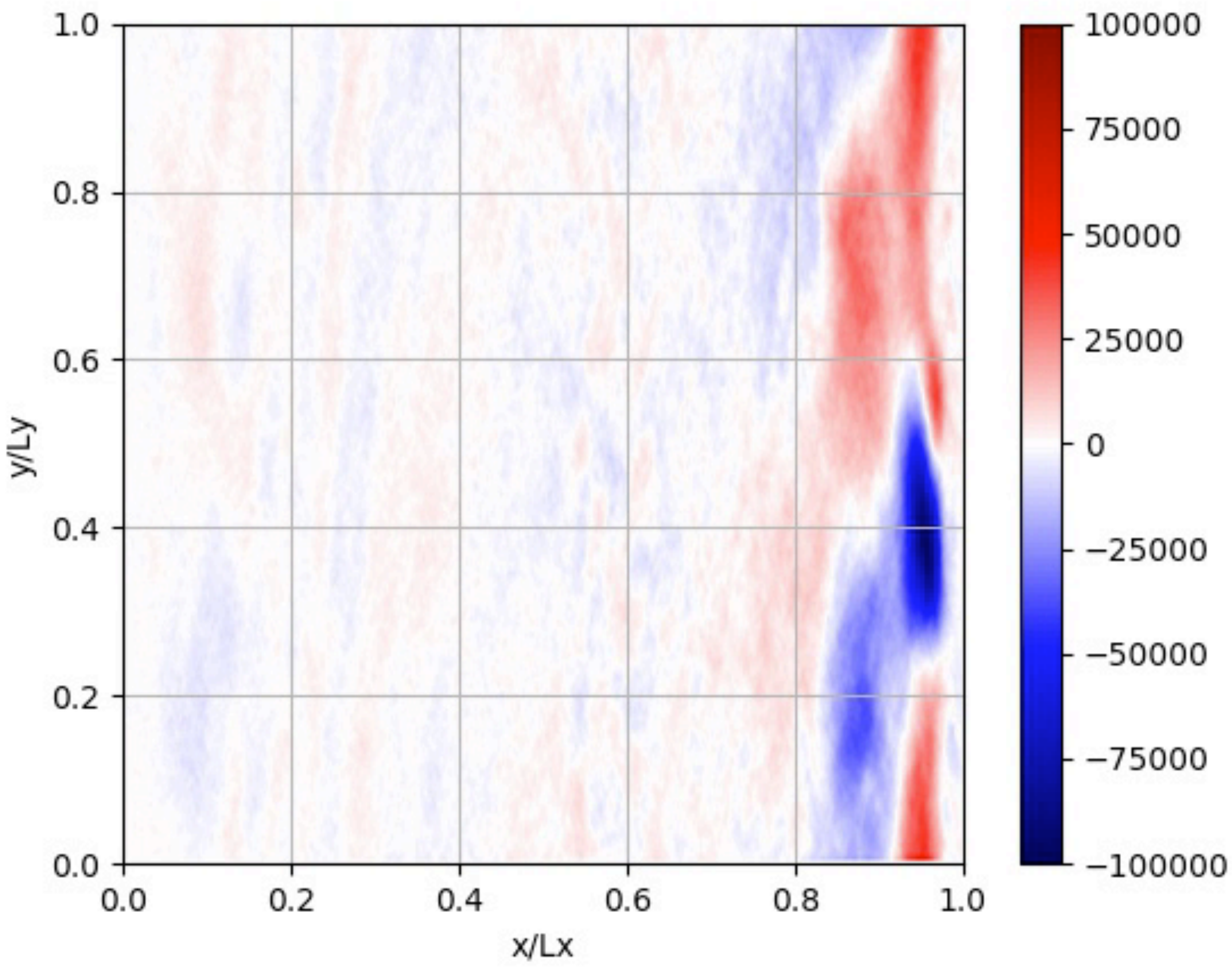
2D "z-θ"





*J.P Boeuf, *Frontiers in Physics* **2**, 74 (2014)

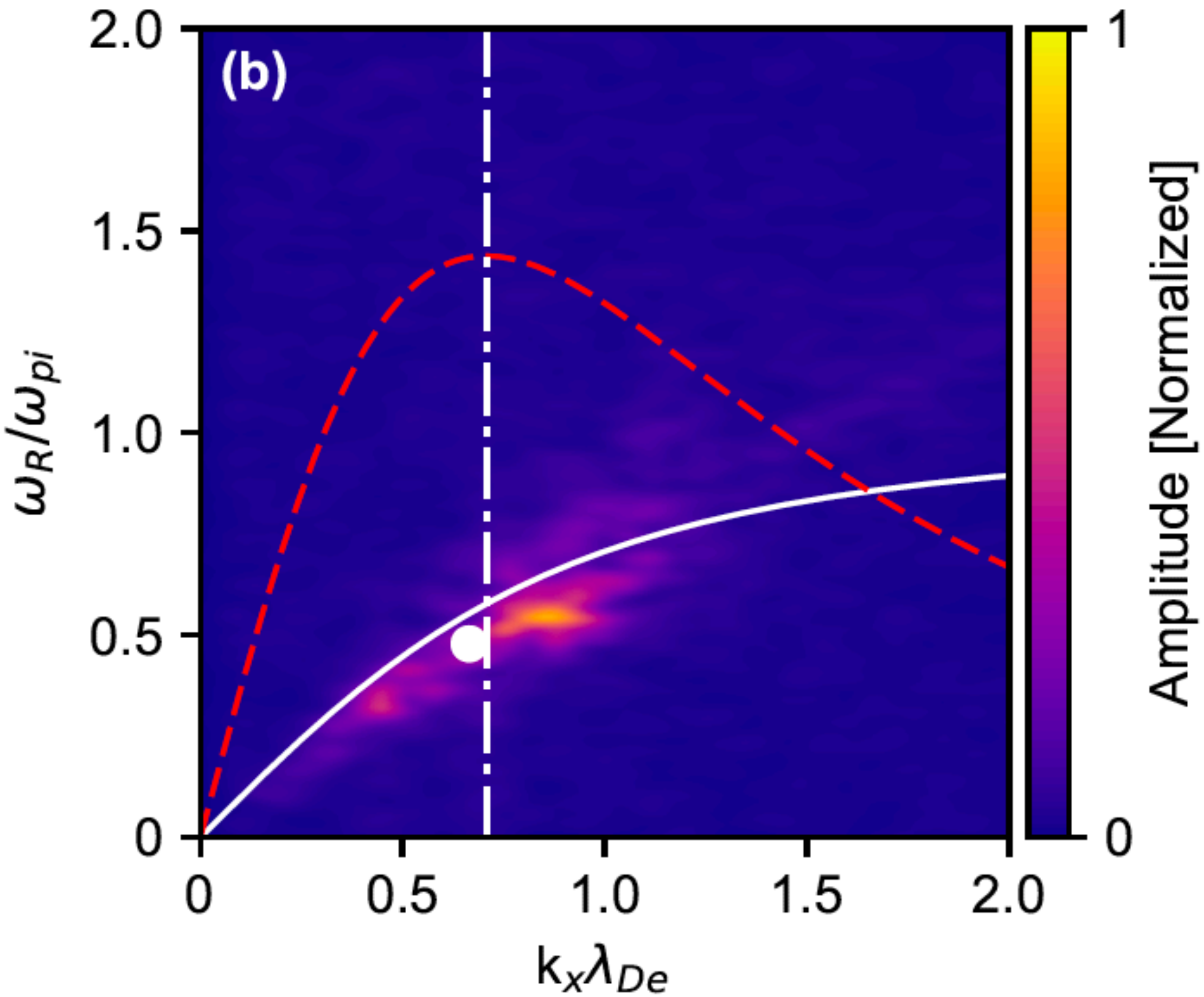
Instability evolution (2D z-θ)



E_θ [V/m]

- Instability wavevector is mainly in the azimuthal direction: except deep inside the thruster where the axial EDI is also important

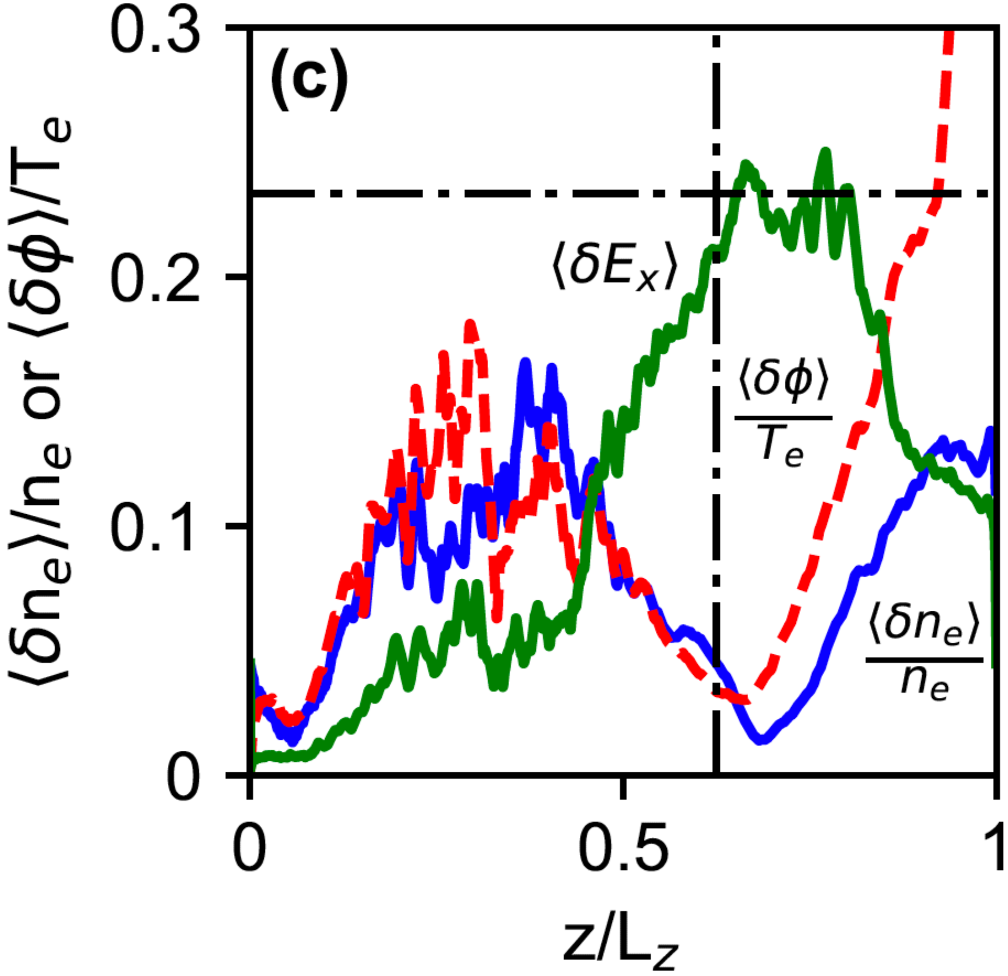
Instability spectrum (2D z-θ)



- Spatio-temporal FFT spectrum matches that for the ion acoustic-type dispersion relation of the EDI, and is similar to experiment*

*S. Tsikata *et al*, *Phys. Plasmas* **17**, 112110 (2010); Lazurenko *et al*, *Phys. Plasmas* 15, 034502 (2008)

Instability amplitude (2D z-θ: at "sat.")



- Amplitude of density fluctuations reaches 1-2% level just downstream of thruster exit: in good agreement with experiment!*

*S. Tsikata *et al*, *Phys. Plasmas* **17**, 112110 (2010)

Electron momentum balance ("theory")

- Complete momentum balance with no assumptions:

$$\frac{\partial}{\partial t}(mn_e\mathbf{v}_e) + \nabla\cdot(mn_e\mathbf{v}_e\mathbf{v}_e) = qn_e(\mathbf{E} + \mathbf{v}_e\times\mathbf{B}) - \nabla\cdot\mathbf{\Pi}_e + \mathbf{R}_{en} + \mathbf{R}_{ei}$$

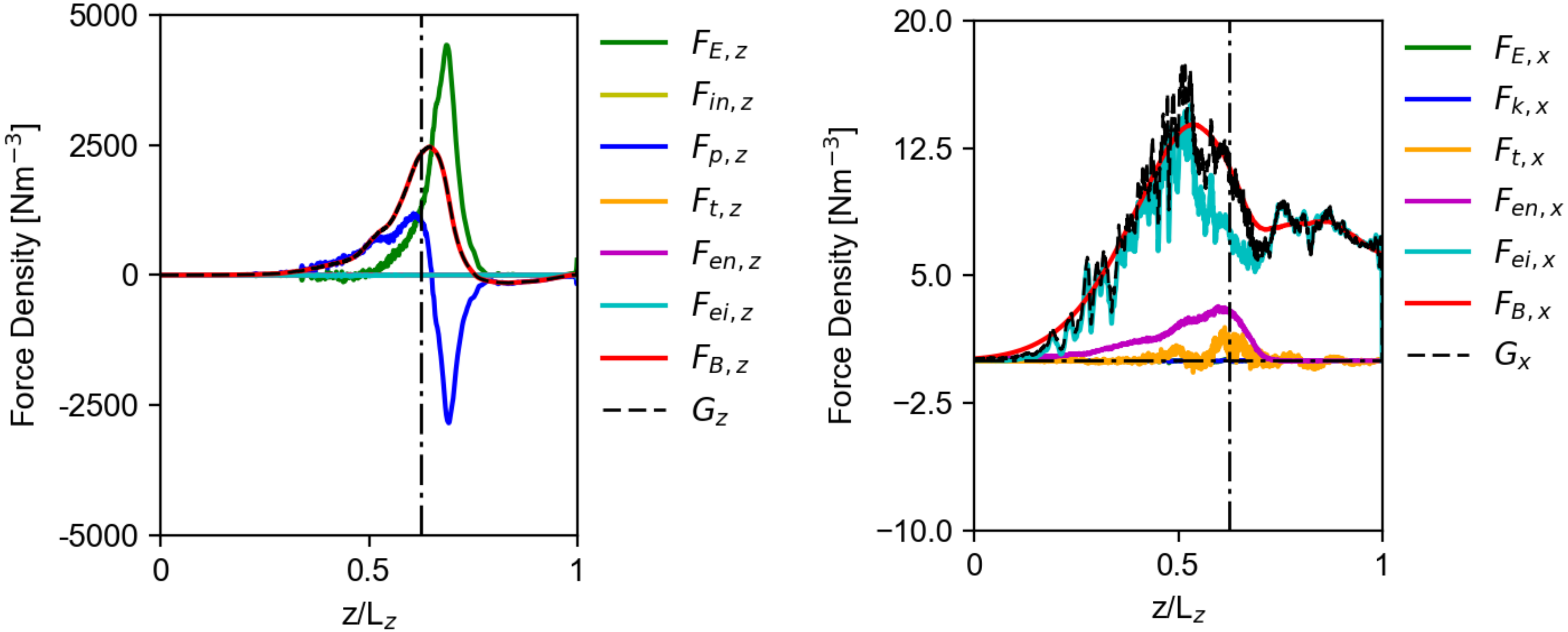
- Average in time and over azimuthal direction. Axial momentum balance is:

$$\underbrace{qn_ev_{ex}B_y}_{F_{B,z}} = \underbrace{\overbrace{\frac{\partial}{\partial t}(mn_ev_{ez})}^{F_{t,z}} + \overbrace{\frac{\partial}{\partial z}(mn_ev_{ez}^2)}^{F_{in,z}} + \overbrace{\frac{\partial\Pi_{e,zz}}{\partial z}}^{F_{p,z}} + \overbrace{(-qn_eE_z)}^{F_{E,z}} + \overbrace{(-R_{en,z})}^{F_{en,z}} + \overbrace{(-R_{ei,z})}^{F_{ei,z}}}_{G_z}$$

- Azimuthal momentum balance is:

$$\underbrace{-qn_ev_{ez}B_y}_{F_{B,x}} = \underbrace{\overbrace{\frac{\partial}{\partial t}(mn_ev_{ex})}^{F_{t,x}} + \overbrace{\frac{\partial}{\partial z}(mn_ev_{ez}v_{ex}) + \frac{\partial\Pi_{e,zx}}{\partial z}}^{F_{k,x}} + \overbrace{(-qn_eE_zx)}^{F_{E,x}} + \overbrace{(-R_{en,x})}^{F_{en,x}} + \overbrace{(-R_{ei,x})}^{F_{ei,x}}}_{G_x}$$

Electron momentum balance (2D r-θ)

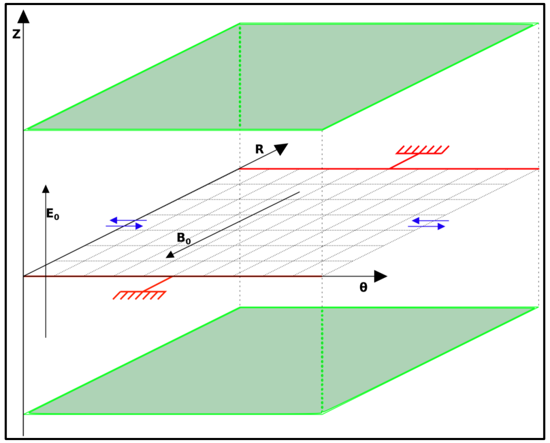


- Electron-ion friction force in PIC found from correlation function:

$$\mathbf{R}_{ei} = q\left\langle \delta n_e \delta \mathbf{E} \right\rangle = q\left\langle \left[\left\langle n_e \right\rangle_t - n_e(t)\right]\left[\left\langle \mathbf{E} \right\rangle_t - \mathbf{E}(t)\right] \right\rangle_t$$

- This friction force completely dominates the azimuthal momentum balance

Parametric dependencies (2D r-θ)



- Instability wavelength and frequency:

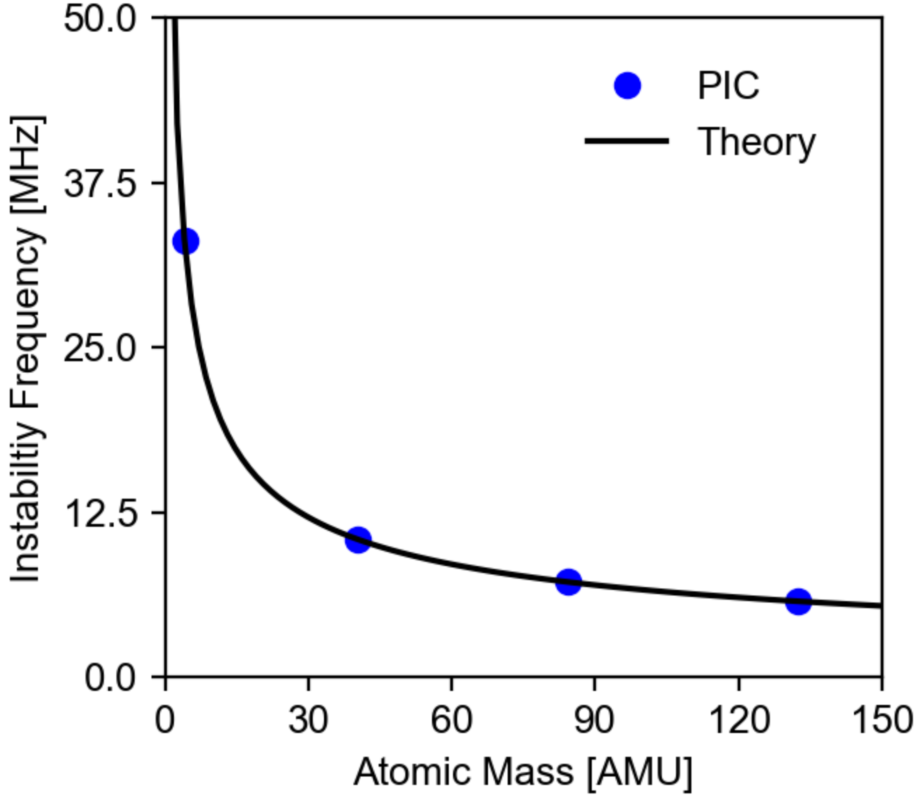
$$\omega_R \approx \frac{\omega_{pi}}{\sqrt{3}} \qquad k\lambda_{De} \approx \frac{1}{\sqrt{2}}$$

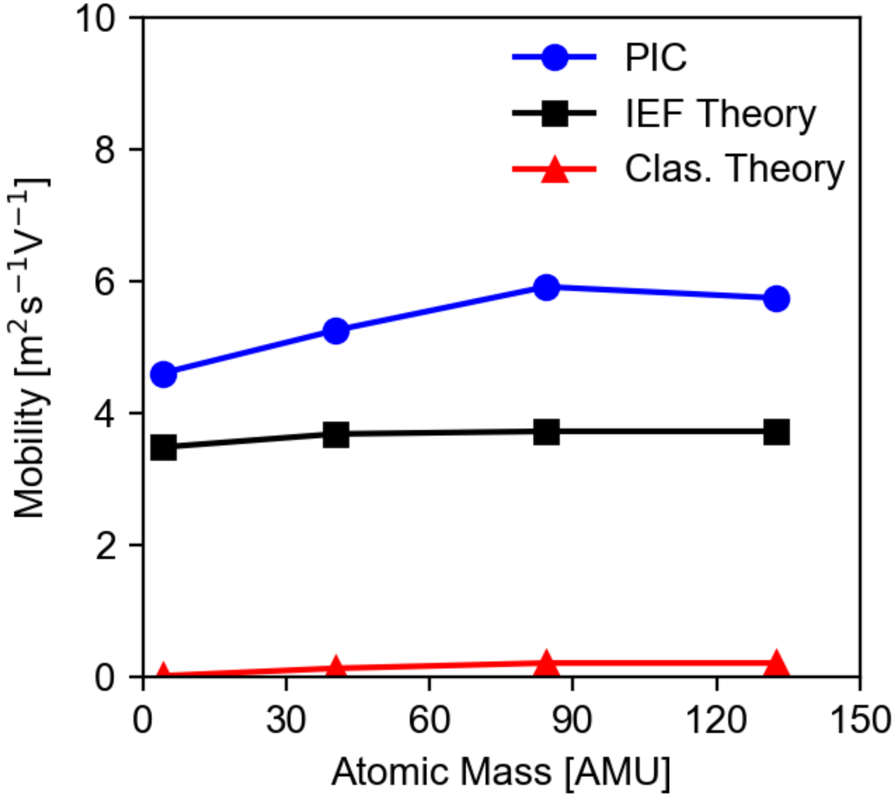
- Friction force and effective mobility:

$$R_{ei}^{IE} \approx \frac{|q|}{4\sqrt{6}} \frac{v_i n_e T_e}{c_s L_z} \quad \Rightarrow \quad \mu_{eff} \approx \frac{\frac{|q|}{m\nu_m}}{1 + \frac{\omega_{ce}^2}{\nu_m^2}} \left[1 + \frac{|q|}{4\sqrt{6} m \nu_m} \frac{B_0}{E_0} \frac{v_i T_e}{c_s L_z} \right]$$

V. Croes *et al*, *Plasma Sources Sci. Technol.* **26**, 034001 (2017)

Ion mass comparison (2D r-θ)



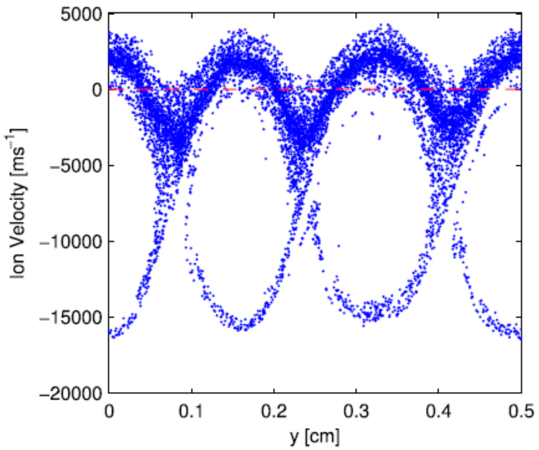


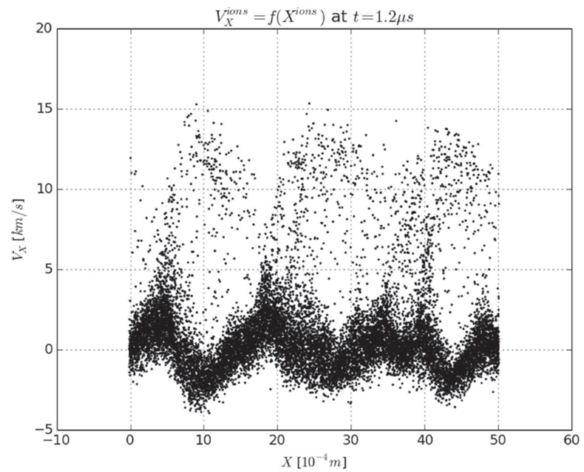
- Similar good agreement obtained for other parameters and instability properties (i.e. wavelength and phase velocity)

- **Secondary electron emission** (not shown) barely affects instability: enhanced friction force still the dominant cross-field transport mechanism

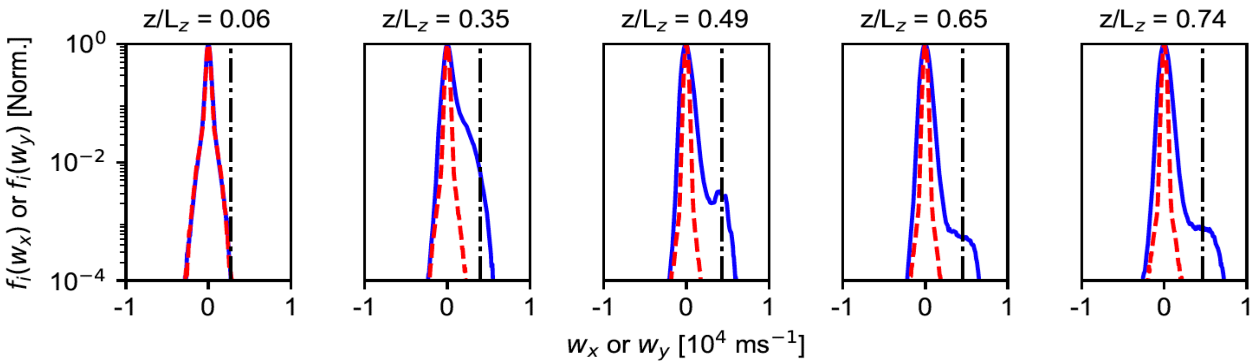
Instability saturation (ion trapping?)

1D "θ"*
2D "r-θ"





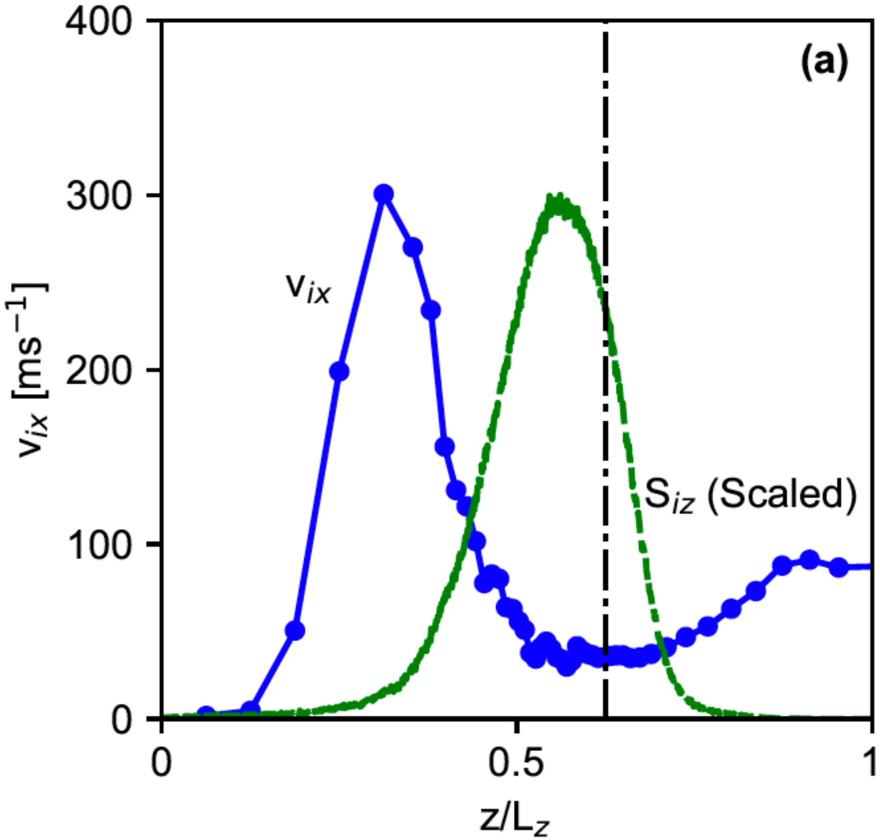
2D "z-θ"

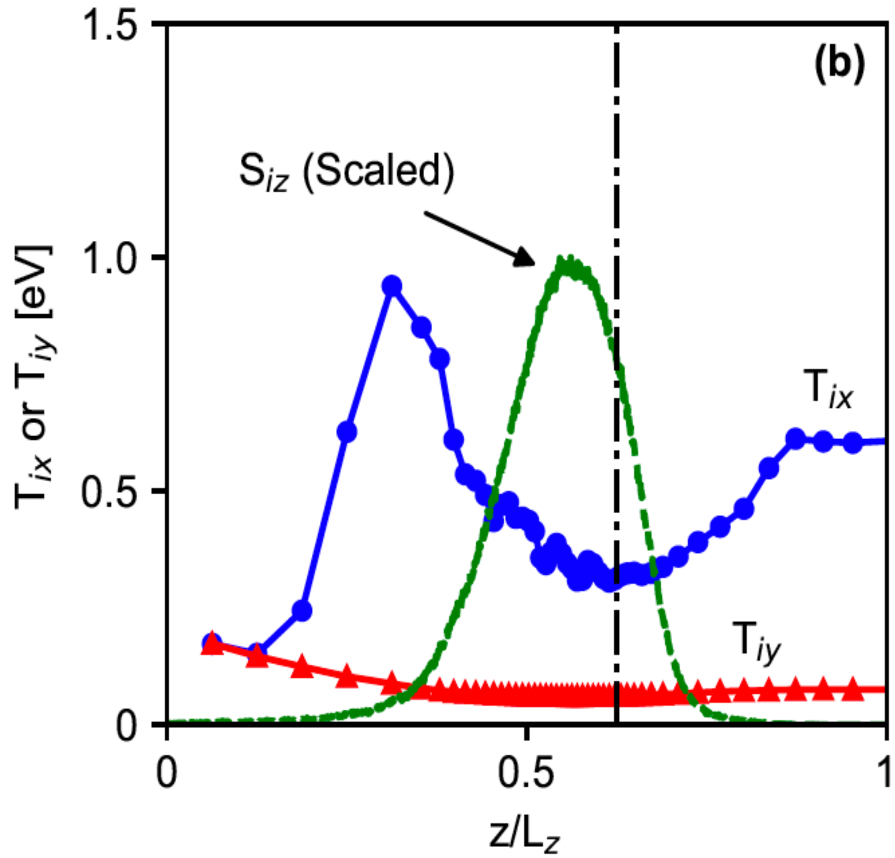


$$\left(v - \frac{\omega}{k_x}\right) \leq 2\sqrt{\frac{|q|\delta\phi}{M}} \quad **$$

*J.P Boeuf, *Frontiers in Physics* **2**, 74 (2014); **A. Degeling and R. Boswell, *Phys. Plasmas* **4**, 2748 (1997)

Ion rotation and heating (2D z-θ)





- Ions rotate in the azimuthal direction with a velocity of the order of the thermal speed, and are heated to almost 1 eV

Summary

- Kinetic theory predicts strong EDIs in typical ExB discharges
- Cyclotron resonance modes tend to be “smeared” out and reduce to “simple” ion acoustic modes
- The EDIs lead to an enhanced electron-ion friction force that acts as an additional momentum loss mechanism increasing cross-field transport
- Ion-wave trapping appears to play an important role in the non-linear evolution and saturation of the instability
- The instability also leads to an ion-electron friction force that causes both ion rotation, and heating
- Predictions from kinetic theory are consistent with experiments and PIC simulations

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