Instabilities and transport in partially magnetized plasmas with ExB drift

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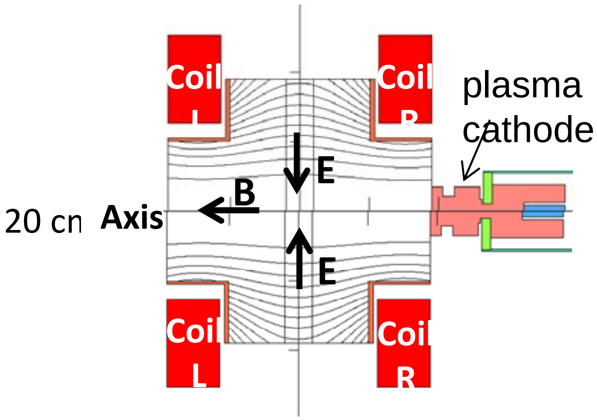
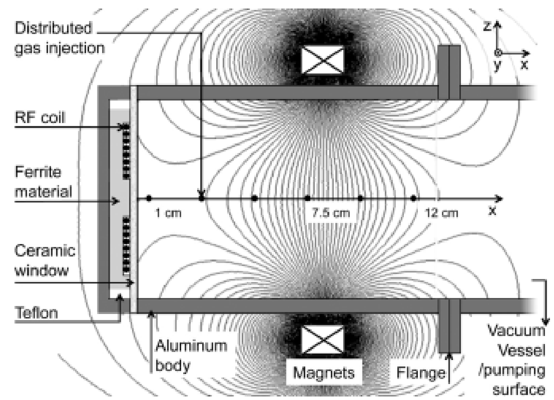
PPPL PRINCETON PLASMA PHYSICS LABORATORY

ExB Workshop , Jun 21st 2017

Common physics in devices of interest: Hall thrusters, magnetrons, Penning sources, magnetic filters (e.g. for negative ion sources), …

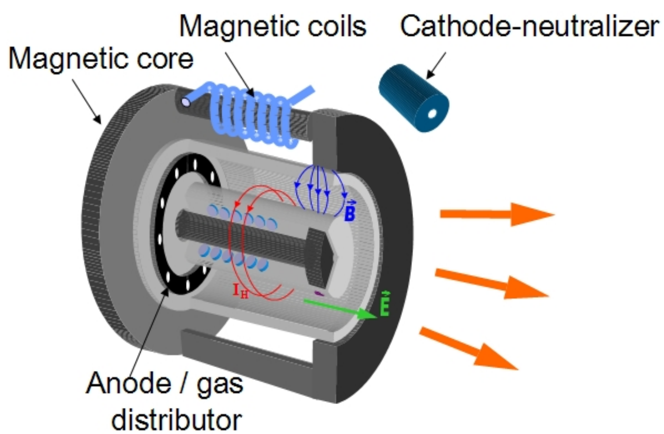


Penning set-up, Y. Raitses PPPL

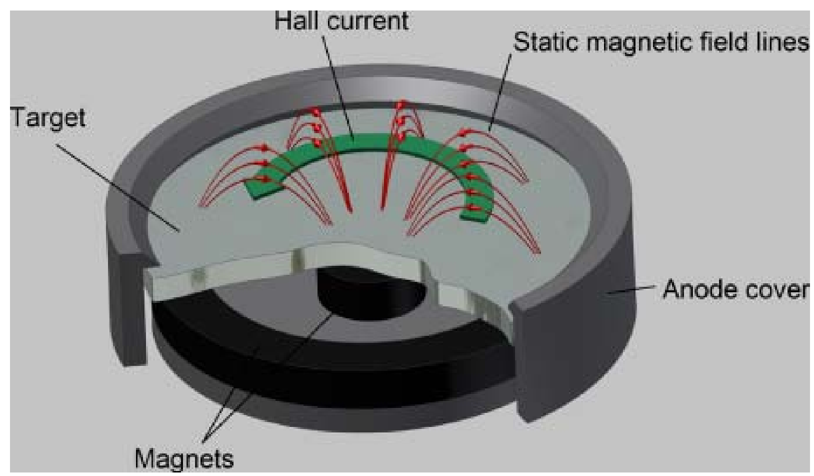


•**Magnetic filter in ICP** ***Aanesland et al., Appl. Phys. Lett. 100 (2012), J.-P. Boeuf (ICOPS 2015)***

Cylindrical magnetron



Y. Raitses et al., J Appl Phys 2000



•**Planar magnetron, J. Winter et al, J. D 2013**

Motivation

- Range of unstable fluctuations of different frequencies and scale ubiquitously observed in Hall thrusters, magnetrons, etc
- Electron transport across the magnetic field is anomalous.
- Large scale coherent structures (spoke)
- Fluctuations across different time and length scales, may have dramatic effects on underlying regimes and performance of various devices
- Nature of fluctuations and transport is poorly understood, precludes first principle predictive modeling

Understanding of physics towards predictive modeling

Overview of instabilities in ExB discharges

- **Basic modes of partially magnetized plasmas**
  - **Ion sound modes**
  - **Anti-drift mode (density gradient effects)**
  - **Lower-hybrid mode (electron inertia)**
- **Destabilization mechanisms**
  - **ExB flow**
  - **ExB flow and dissipation (collisions and ionization)**
  - **Finite length/boundaries/sheath effects**
  - **Kinetic/resonance effects (Electron cyclotron drift instability S Janhunen et al., ArXive:1705:00749, 2017)**
- **Finite length/boundaries as an instability mechanism**
  - **Kapulkin TPS IEEE 2015, Koshkarov et al PoP 2016**
- **Ionization modes, comment on predator-prey**
- **3D and sheath boundaries effects**
- **Summary**

Comments on large scale (quasi-coherent) structures (e.g. spokes and breathing mode) and other important items

- Large scale modes: primary (linear) instability or nonlinear drive/self-organization from other (small scale) modes –inverse cascade?
- Coexistence of large and small scale modes
  - Contributions to anomalous transport
- Ionization effects:
  - Predator-prey models?
  - Importance of the underlying instability!
- Nontrivial effects related to the electron dynamics along the magnetic field

Methodology

- Fluid theory models: reduced electron dynamics and advanced electron FLR model ($\omega << \omega_{ce}, k_{\perp}\rho_e > 1$), full unmagnetized ions, …
- Analysis of linear eigen-modes and instabilities (local and nonlocal analysis, dispersion relation solver)
- Nonlinear simulations within the BOUT++ framework adapted for partially magnetized plasmas, Generic rectangular geometry, 3D in development…
- Inspired and guided by the input from the experiments and PIC at PPPL; numerical parameters are those of CHT and Penning discharge (PPPL)

Also, Nonlinear fluid simulations with MAGNIS,
presentation by G Hagelaar,
and a poster by S. Sadouni, G. Hagelaar,

Eigen-modes of partially magnetized plasmas

- Ion sound waves $\omega^2 = k^2 c_s{}^2$
- Anti-drift modes $\omega = \dfrac{k^2 c_s{}^2}{\omega_*}$
- Lower- hybrid modes $\omega = \sqrt{\omega_{ci}\omega_{ce}}$

Free energy sources destabilizing the modes

- Electron ExB drift flow
- Ion flow across the magnetic field
- Gradients of density, electron temperature and magnetic field (perpendicular to the magnetic field)
  – Dissipation (collisions, ionization) are often destabilizing for negative energy modes in systems with flows
  – Small scale transport may work as an anomalous conductivity triggering the dissipative modes

	Electron flow $E_0 \times B$	Ion flow V_{i0}	Density gradient ∇n	electron inertia	e-N coll.	i-N coll.
Electron flow $E_0 \times B$						
Ion flow V_{i0}						
Density gradient ∇n						
electron inertia						
e-N coll						
i-N coll.						

- Collisional Simon-Hoh
- Farley-Buneman instability*
- Gradient-drift instability*
- Collision-less Simon-Hoh
- Anti-drift mode
- ….

Ionosphere irregularities, E layer, affecting communications, GPS, etc; have been studied experimentally and theoretically since 60s, no accepted nonlinear theory, only partial agreement with linear theory

Collisionless Simon-Hoh (Sakawa, Chen 1992) or destabilized anti-drift mode (Friedman, 1964)

- Finite density gradient, $\nabla n_0 \neq 0$
- Finite electric field, $E_0 \neq 0$
- Finite magnetization of electrons
- Ions are not magnetized
- No electron inertia
- Ion inertia
- No collisions

"Anti-drift" mode + ExB flow: Collision-less Simon-Hoh instability

- Gradient effects in electron dynamics

$$-i\omega n_e + \tilde{V}_E \cdot \nabla n_0 = 0 \qquad \frac{n_e}{n_0} = \frac{\omega_*}{\omega}\frac{e\phi}{T_e} \qquad \omega_* = \frac{kcT_e}{eB}\left(\frac{\nabla n}{n}\right)^{-1}$$

- Unmagnetized Ions

$$-i\omega m_i V_i = eE$$
$$-i\omega n_i + \nabla \cdot (n_i V_i) = 0 \qquad n_i = \frac{e\phi}{T_e}\frac{k^2 c_s^2}{\omega^2} n_0$$

$$\omega = \frac{k^2 c_s^2}{\omega_*}$$

- Anti-drift mode (Fridman, 1964)
- Collisionless Simon-Hoh instability $\omega \to \omega - \omega_E$

$$\omega_* / \omega_E > 0 \quad E \cdot \nabla n_0 > 0$$

$$\frac{k_\theta^2 c_s^2}{\omega^2} = \frac{\omega_*}{\omega - \omega_E}$$

Doppler shift

$$\gamma \propto kc_s \sqrt{\frac{L_n}{L_E}}$$

What happens at high k?

Sakawa, Chen 1992

Lower-hybrid modes destabilized by density gradient

- Finite density gradient, $\nabla n_0 \neq 0$
- Finite electric field, $E_0 \neq 0$
- Finite magnetization of electrons;
- ions are not magnetized
- Electron inertia is included
- Ion inertia is included
- No collisions

Destabilized lower hybrid modes

- Anti-drift mode, magnetic field gradients
  - Hall thruster plasmas: Esipchuk, Tilinin 1973, Kapulkin, 1995, Litvak, Fish 2001, Frias, Smolyakov, 2013,2014
- Electron inertia lower-hybrid modes $\omega_{LH}^{2} = \omega_{ce}\omega_{ci}$

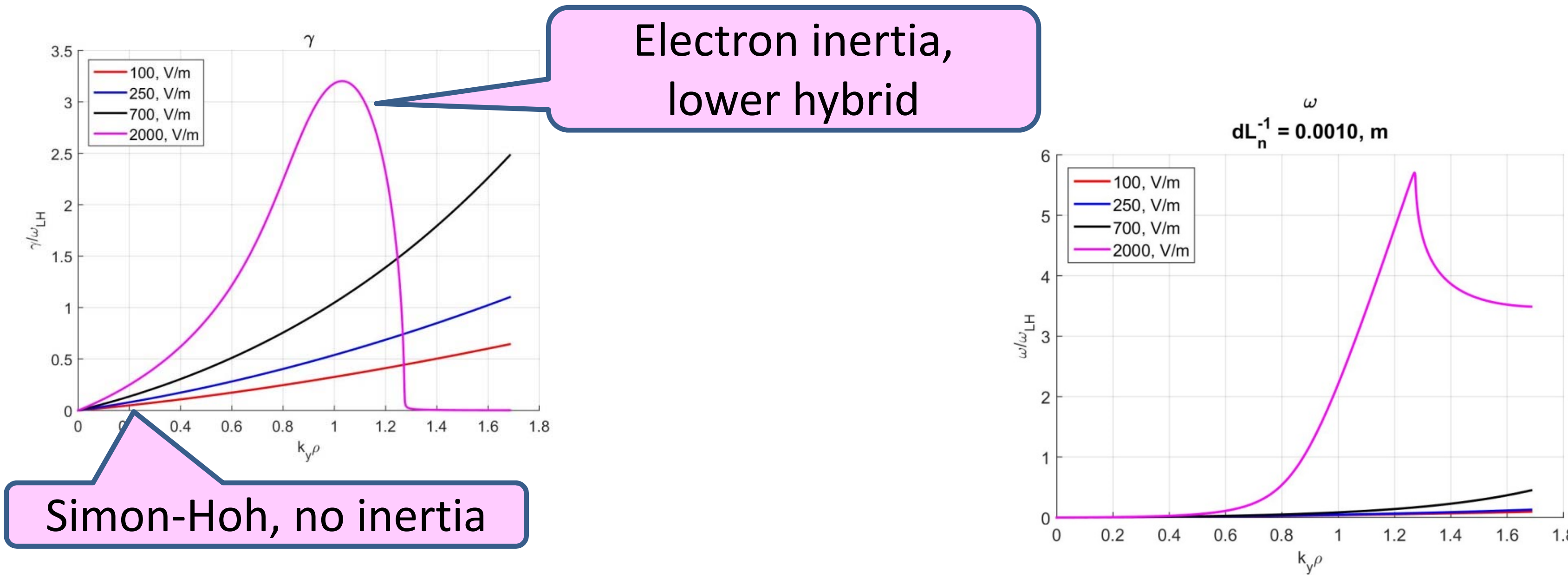
$$k^2 \lambda_{De}^{2} - \frac{k^2 c_s^{2}}{\omega^2} + k^2 \rho_e^{2} + \frac{\omega_* - \omega_D}{\omega - \omega_E - \omega_D} = 0$$

Ion inertia | Electron inertia | Anti-drift

- Lower hybrid modes (important at smaller scales) are destabilized by gradients, shear flows, collisions

Turbulent heating in shock waves, Krall, Liewer, 1971; Davidson, Krall 1971; Sagdeev 1968; Alikhanov 1968

Lower-hybrid modes destabilized by density gradient; continuation of the collisionless Simon-Hoh



- Finite density gradient,
- Finite electric field,
- Finite magnetization of electrons;
- Ions are not magnetized
- Electron inertia is included
- Ion inertia is included
- No collisions

Some comments on transport: Mixing length estimates

$$D_{GB} = \frac{(\Delta x)^2}{\tau} = \frac{\gamma}{k_\perp^{\ 2}}$$

Large scale dominant

Gyro-Bohm: fully (electron and ions) magnetized plasma

$$k_\perp^{\ 2} \rho_i^{\ 2} \approx 1 \qquad \gamma \approx \omega_*$$

$$D_{GB} = \frac{\gamma}{k_\perp^{\ 2}} = \omega_* \rho_i^{\ 2} = v_* \rho_i = \frac{\rho_i}{L_n} \frac{cT}{eB}$$

Bohm diffusion: strong turbulence, demagnetized ions,

$$\gamma \approx \omega_{ci} \quad k_\perp \approx \rho_i^{\ -1}$$

$$D_B = \frac{\gamma}{k_\perp^{\ 2}} = \omega_{ci} \rho_i^{\ 2} = \frac{cT}{eB}$$

Simon-Hoh and lower hybrid

$$\gamma \propto k c_s \sqrt{\frac{L_n}{L_E}}$$

$$D_B = \left(\left(\frac{m_e}{m_i} \right)^{1/2}, \left(\frac{L_n}{L_E} \right)^{1/2} \right) \frac{cT}{eB}$$

$$\gamma \propto \omega_{LH} = \sqrt{\omega_{ci} \omega_{ce}}$$

Complex picture of instabilities in inhomogeneous partially magnetized plasmas:

Some less trivial/known situations

-short wavelength ion-sound destabilized by collisions
-anti-drift modes modes driven by ion beam alone (no ExB drift)

Low and high frequency ion sound modes

- Ions: $$-i\omega m_i V_i = eE$$ $$-i\omega n_i + \nabla\cdot\left(n_i V_i\right) = 0$$ $$n_i = \frac{e\phi}{T_e}\frac{k^2 c_s^{\;2}}{\omega^2} n_0$$
- "Low frequency" ion sound $$\omega < k_\parallel v_{Te} \qquad n_e = \frac{e\phi}{T_e} n_0$$
  - Electron dynamics , finite $k_\parallel$
- "High frequency" ion sound, large $k_\perp \rho_e >> 1$

Short wavelength extension of the lower-hybrid mode

$$\omega^2 = \omega_{LH}^{\;\;2}\left(1 + k_\perp^{\;2}\rho_e^{\;2}\right) \approx \omega_{LH}^{\;\;2} k_\perp^{\;2}\rho_e^{\;2} = k_\perp^{\;2} c_s^{\;2}$$

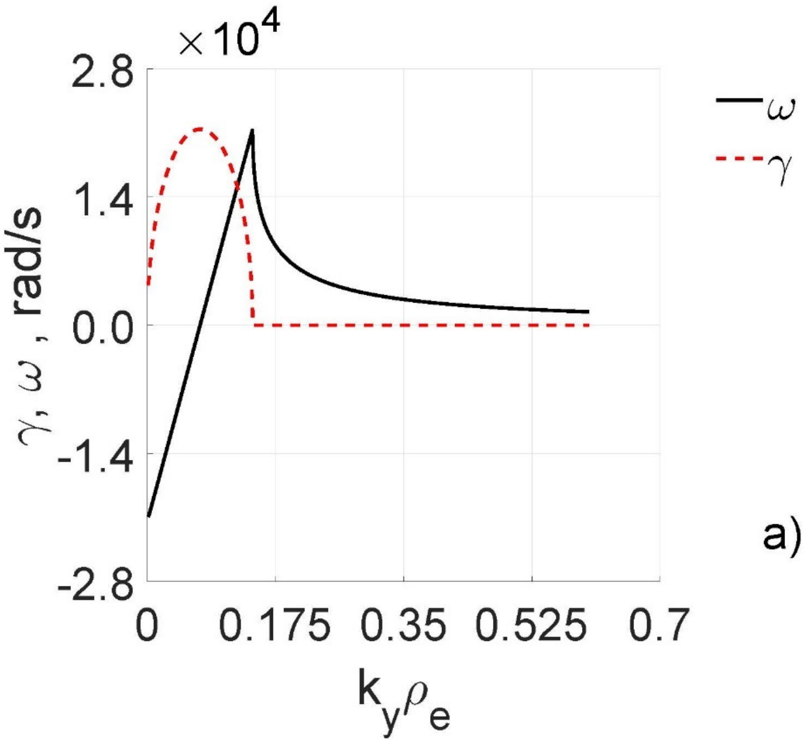
Demagnetized electrons for

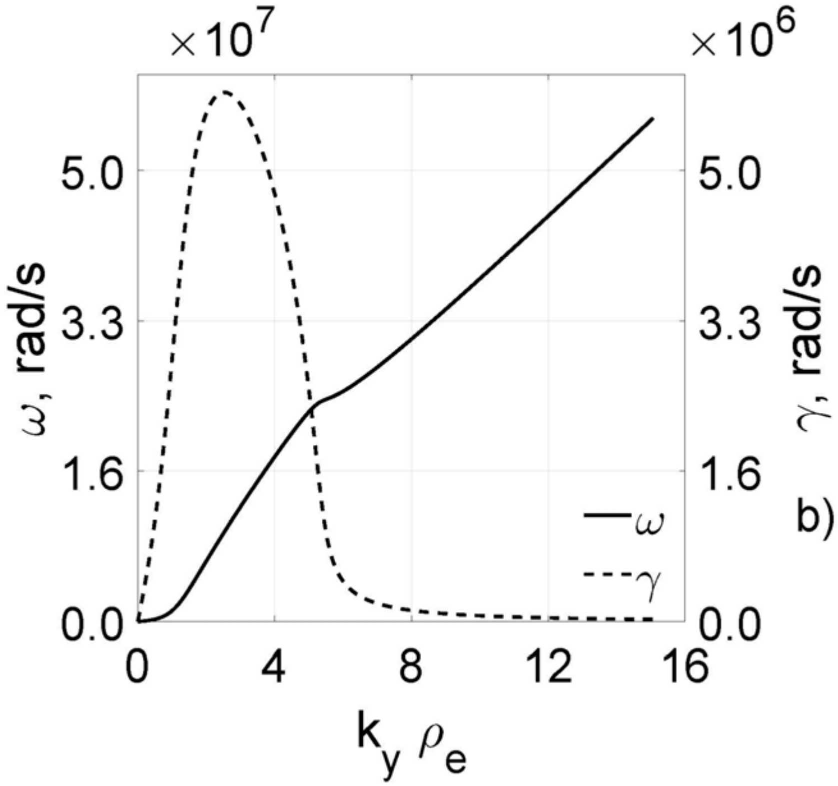
$$k_\perp \rho_e >> 1$$

$$n_e = \frac{e\phi}{T_e}\left(1 - I_0\left(k_\perp^{\;2}\rho_e^{\;2}\right)\exp\left(-k_\perp^{\;2}\rho_e^{\;2}\right) + 2\left(\omega - k_\perp v_E\right)^2 \sum_{m=-\infty}^{m=\infty} \frac{I_m\left(k_\perp^{\;2}\rho_e^{\;2}\right)\exp\left(-k_\perp^{\;2}\rho_e^{\;2}\right)}{\left(\omega - k_\perp v_E\right)^2 - m^2\omega_{ce}^{\;\;2}}\right)$$

$$I_m\left(k_\perp^{\;2}\rho_e^{\;2}\right)\exp\left(-k_\perp^{\;2}\rho_e^{\;2}\right) \to 0$$

Complex picture of instabilities in inhomogeneous partially magnetized plasmas, e.g.





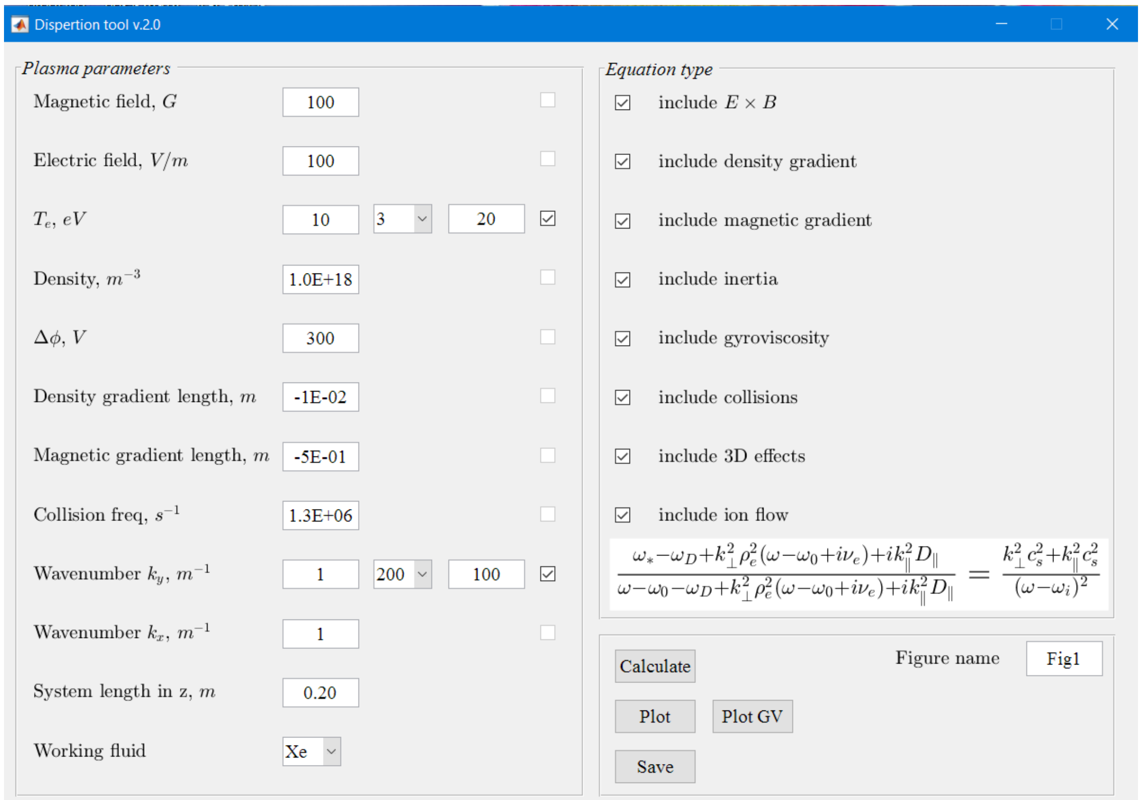
Anti-drift mode destabilized by the ion beam velocity and density gradient, E=0

Ion sound mode destabilized by the density gradient and collisions, E=0

Smolyakov et al., PPCF 2016

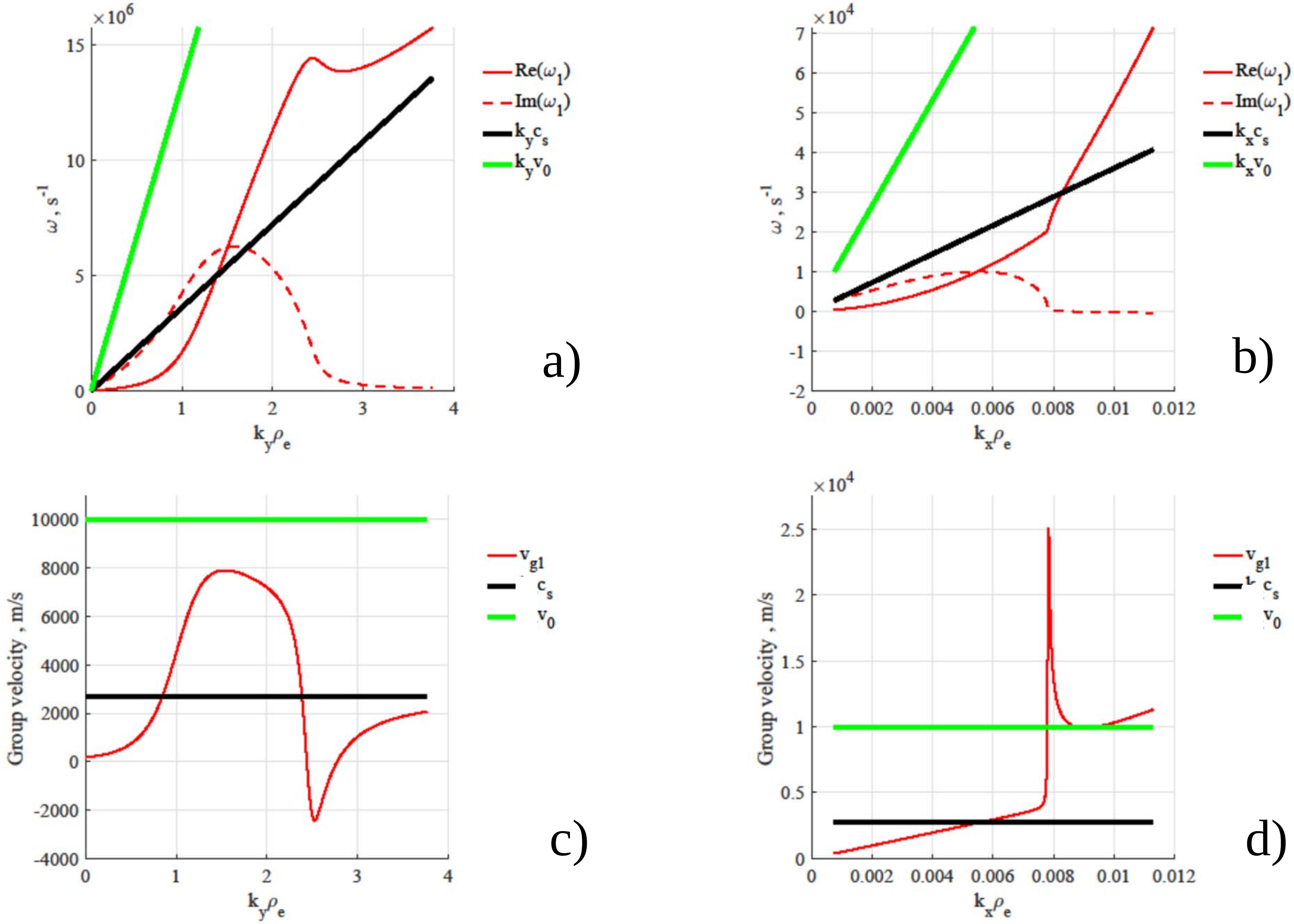
MATLAB solver for general fluid modes dispersion relation

I. Romadanov et al.,
arXiv.org > physics > arXiv:1610.00218
https://bitbucket.org/ivr509/hall-plasmas-discharge-solver/downloads/



- Change the form of the dispersion relation by adding different physical effects.
- Set any physical parameter (*B*, *E*, *T*, etc.) as a variable
- Investigate unstable modes in azimuthal (*y*) or axial (*x*) directions.
- 2D picture of instabilities in k_y-k_x space.

Examples of output



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Locating the Zeros of an Analytic Function

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Received October 22, 1984; revised September 25, 1985

A numerical technique is presented for locating the zeros of an analytic function in the complex plane. The methods used are not new: the important content of this paper is the development and testing of a method to a point where it may be used with confidence and reliability. An application considered here is the location of the eigenvalues of the Orr–Sommerfeld equation for plane Poiseuille flow in a specified portion of the complex (eigenvalue) plane.

ZEAL: A mathematical software package for computing zeros of analytic functions

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Received 3 December 1998; accepted 28 June 1999

Abstract

We present a reliable and portable software package for computing zeros of analytic functions. The package is named ZEAL (ZEros of AnaLytic functions). Given a rectangular region W in the complex plane and a function $f : W \to \mathbb{C}$ that is analytic in W and does not have zeros on the boundary of W, ZEAL localizes and computes *all* the zeros of f that lie inside W, together with their respective multiplicities. ZEAL is based on the theory of formal orthogonal polynomials. It proceeds by evaluating numerically certain integrals along the boundary of W involving the logarithmic derivative f'/f and by solving generalized eigenvalue problems. The multiplicities are computed by solving a linear system of equations that has Vandermonde structure. ZEAL is written in Fortran 90.

Nonlinear simulations with BOUT*++

- High performance platform for fluid and plasma simulations in vector form in 3D curvilinear magnetic field geometry
- Open magnetic field line configurations are included, allows sheath boundary conditions
- Designed and tested with reduced plasma fluid models applications in mind (e.g. Poisson brackets for nonlinear terms, Arakawa type schemes)
- Initial value, finite-difference and Fourier
- Widely used for edge tokamak plasmas

*. D. Dudson, M. V. Umansky, X. Q. Xu, P. B. Snyder and H. R. Wilson Computer Physics Communications 2009; BOUT++: A framework for parallel plasma fluid simulations

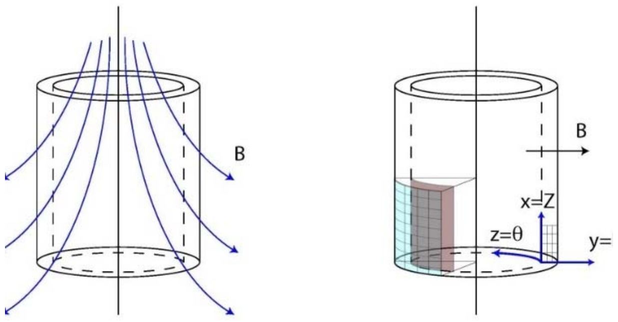
First principle fluid simulation of turbulence and transport

-2D domain (periodic and axial)

-Gradient driven turbulence simulation:

$$n(z,t)=n_0(Z,T)+\tilde{n}(z,t) \qquad n_0(Z),\Phi_0(Z),T(Z)$$

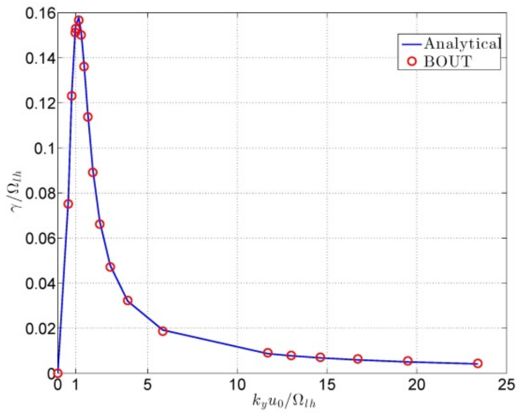
Hall thruster schematic and computational grid

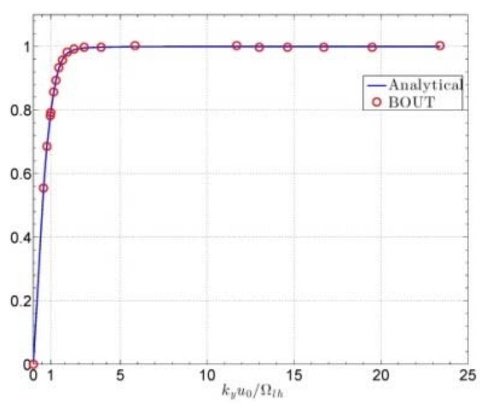


B=[grad(z), grad(x)]

Benchmarks:

- Linear initial value dynamics is tested against the linear eigen-value solutions
- Grid convergence
- Nonlinear saturation, energy balance





Good agreement between BOUT initial value and eigen-value solvers

Reduced nonlinear equations: $\omega_{ce}^{-1}\partial/\partial t << 1$

Density gradient, Simon-Hoh

Magnetic field gradient

Collisions

Electrons

$$\frac{\partial n}{\partial t}+\frac{c}{B}\{\phi,n\}+\mathbf{v}_E\cdot\nabla n_0-2n\mathbf{v}_E\cdot\nabla\ln B-2n\mathbf{v}_{pe}\cdot\nabla\ln B+\frac{\nu c n_0}{B_0\omega_{ce}}\nabla_\perp^2\left(\phi-\frac{T_e}{en_0}n\right)$$

$$+\frac{n_0 c}{\omega_c B_0}\left(\frac{\partial}{\partial t}+\mathbf{v}_E\cdot\nabla\right)\nabla_\perp^2\phi+\frac{c^2T_e}{eB^2\omega_{ce}}\nabla\cdot\{\nabla\phi,p\}=0$$

inertia

Ion flow

Ions

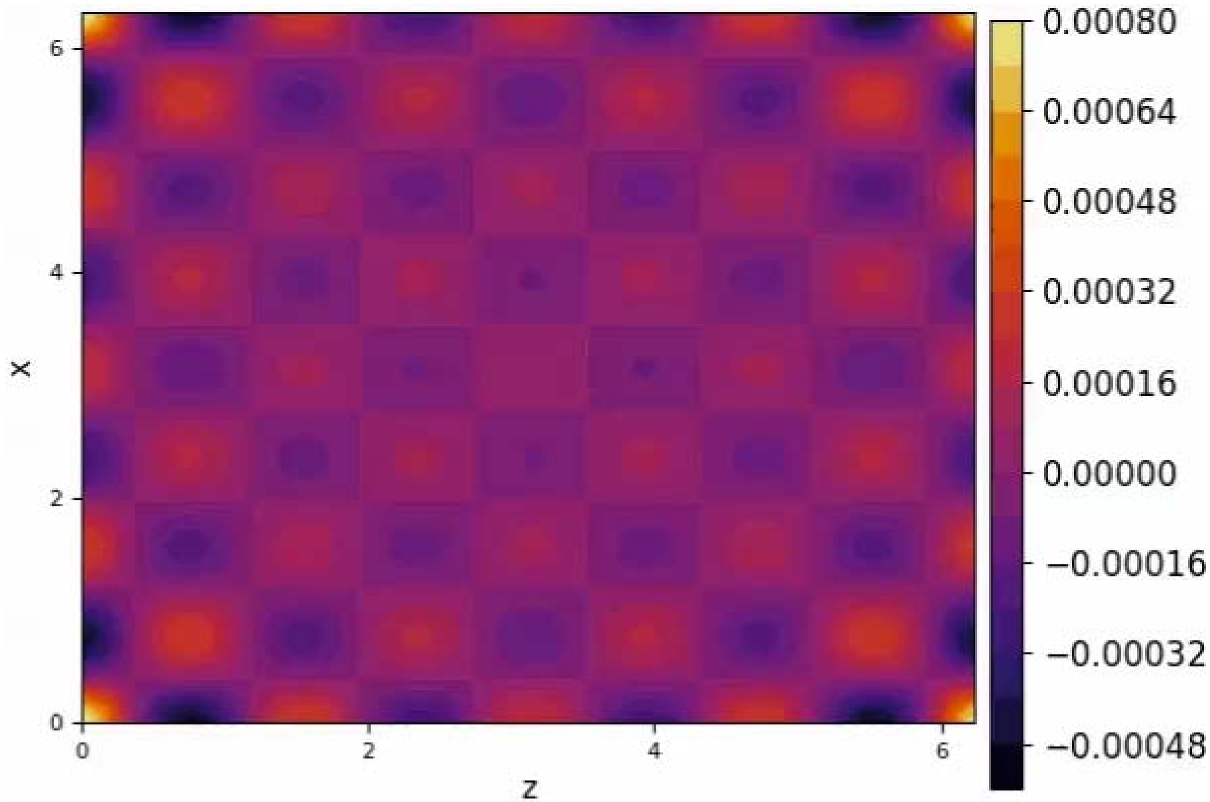
$$\tilde{\mathbf{v}}=-\nabla\tilde{\chi}.$$

$$\frac{\partial}{\partial t}\nabla^2\tilde{\chi}=\nabla^2\left(\frac{e}{m_i}\phi+\frac{(\nabla\tilde{\chi})^2}{2}\right)-v_0\frac{\partial}{\partial z}\nabla^2\tilde{\chi}-2\frac{\partial v_0}{\partial z}\frac{\partial^2\tilde{\chi}}{\partial z^2}-\frac{\partial\tilde{\chi}}{\partial z}\frac{\partial^2 v_0}{\partial z^2}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_0\cdot\nabla\right)\tilde{n}-\nabla\tilde{\chi}\cdot\nabla n_0+\tilde{n}\nabla\cdot\mathbf{v}_0-n_0\nabla^2\tilde{\chi}-\nabla\cdot(\tilde{n}\nabla\tilde{\chi})=0.$$

2D: azimuthal-axial for thruster and azimuthal-radial for Penning

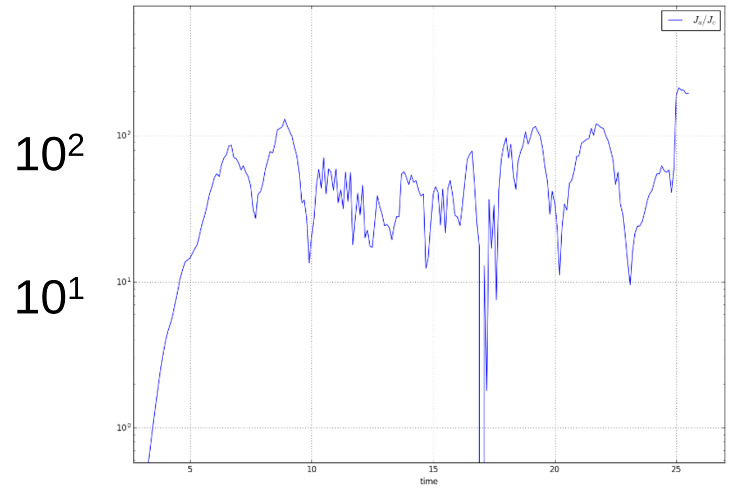
Vorticity structure



Transport and structures

- Highly intermittent turbulent current

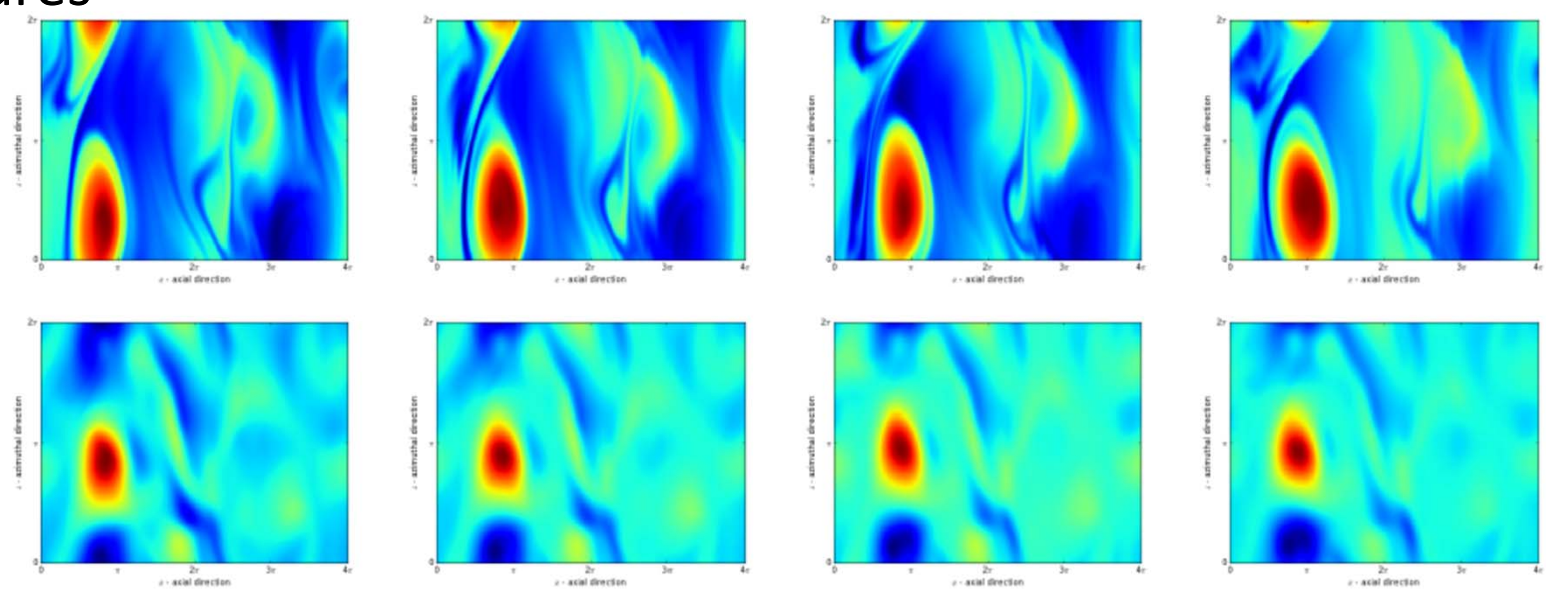
Effective Hall parameter ~27-54



- There is correlations between vorticity and anomalous current structures

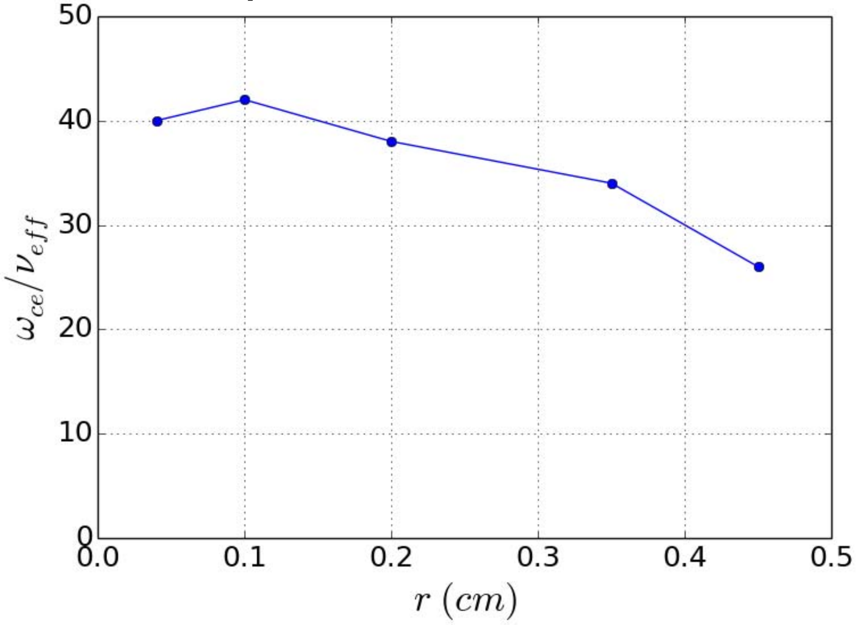
vorticity

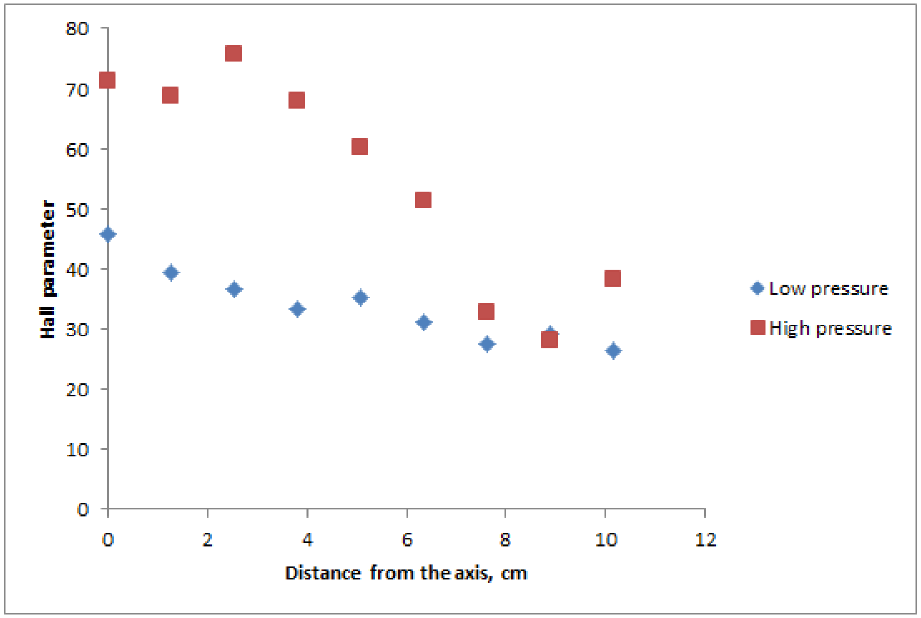
anomalous current



Anomalous transport in experiment and PIC simulations

Hall parameter in simulations (left) and experimental data (right). B=100G, P=0.2mTorr.





2D PIC simulations Calrsson, Kaganovich et al IEPC 2015

Penning discharge experiment, Raitses et al, IEPC 2015

Some conclusions from nonlinear simulations

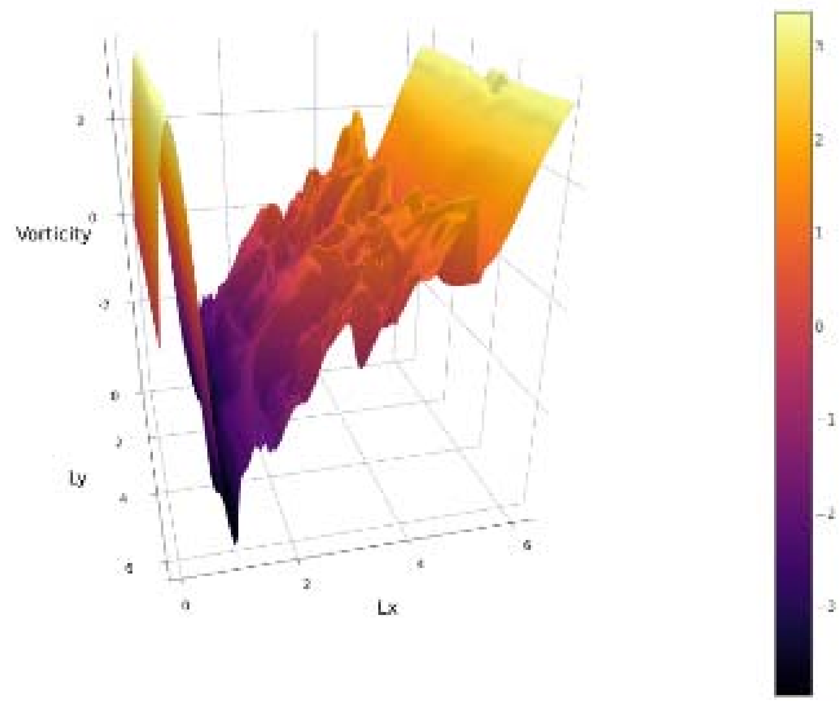
- Nonlinear fluid simulations of inhomogeneous partially magnetized plasma with electron and ion beams, and collisions demonstrate:
  - Significant anomalous current (highly intermittent)
  - Tendency toward formation of large scale structures (inverse cascade)
    - Analytical theory has been proposed to explain it as a result of modulational instability from the bath of unstable small scale mode, Lakhin et al, PoP 2016
  - Correlation between large scale structures and enhanced density of the anomalous current

Closer look into nonlinear simulations and theory

Some slowly growing modes have been detected in the simulations

- Large scale
- Slow growth rate but hard to stabilize
- Large amplitude
- Background for other small scale modes

2D azimuthal-axial simulations



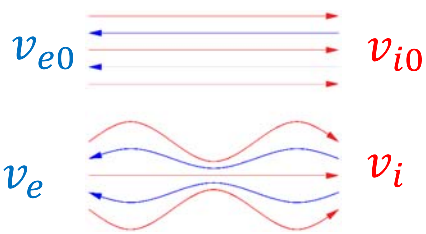
Resistive current flow instabilities

- No density gradient required
- Occur in the direction of the current flow
- Either along the stationary ExB flow or along the ion flow

Fish, Litvak 2001
Chable, Rogier, 2005
Fernandez et al 2008
Koshkarov et al, 2017

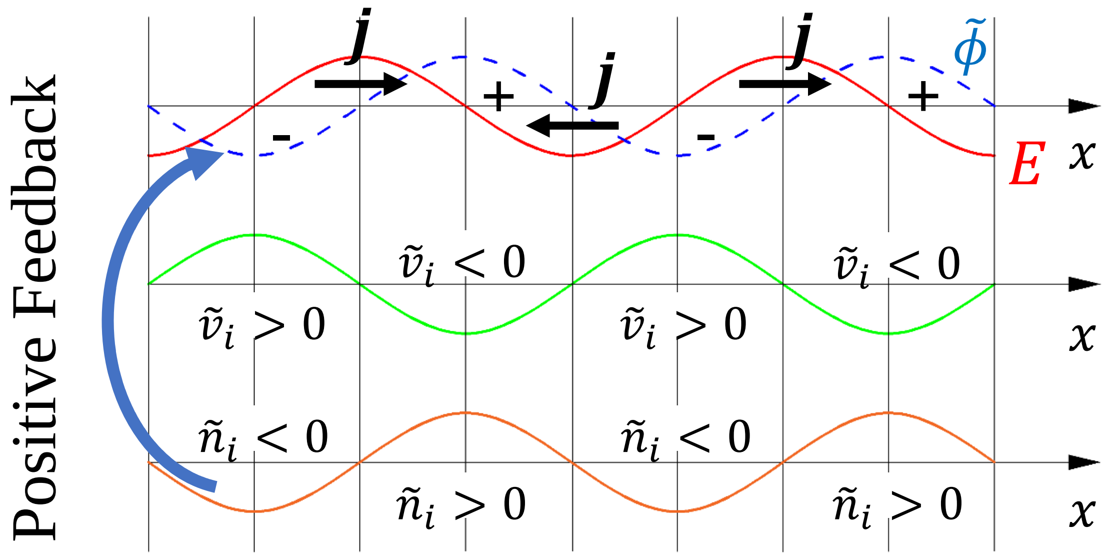
In case of the are axial current (ion) flow is an important ingredient of breathing mode oscillations? Chable, Rogier 2005

Axial-flow instability and large scale structures



Inherent instability of the current flow: Dissipative response of the electron flow + ballistic (inertial) response of ions

Electron current:
collisional and/or anomalous



$$J = \sigma_{a,c} E$$

$$\frac{m_i v^2}{2} + e\phi = const$$

$$\tilde{v} = -\frac{e\tilde{\phi}}{m_i v_{i0}}$$

$$nv = const$$

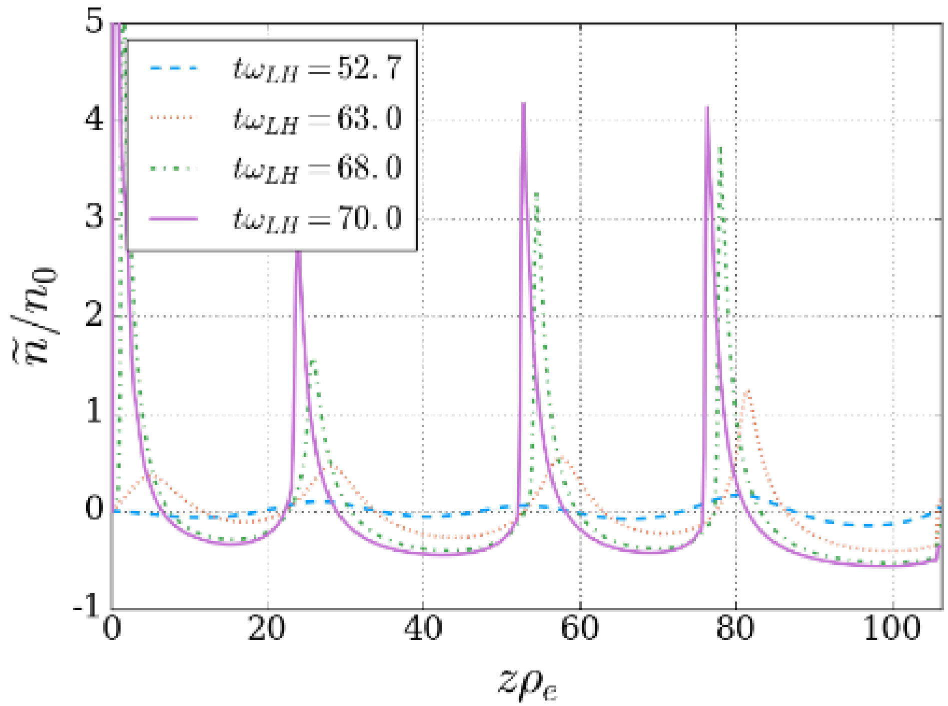
$$\tilde{n} = -\frac{\tilde{v}_i n_0}{v_{i0}} = -\frac{en\tilde{\phi}}{m_i v_{i0}}$$

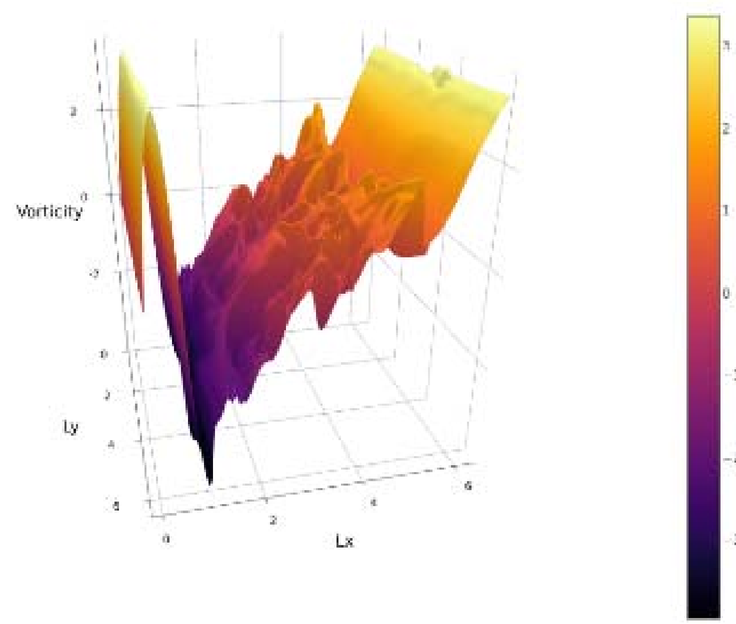
Koshkarov et al, PoP 2017 to appear

Ion response is inertial: in phase with potential

Axial-flow instability and large scale structures

- Axial mode are also found in nonlinear 2D simulations, coexist with small scales (lower hybrid modes)
- Weakly growing structures, hard to saturate, large amplitude, slowly moving, resemble non-monotonous structures in the electric field (Vaudolon, Khiar, Mazouffre 2014)
- Important for ionization (breathing) modes?





Moving toward the spoke modeling: ionization effects

Ion-neutrals coupling: comment on the predator-prey model

Continuous (differential) model:

Ions: $$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = k_i n N,$$

neutrals: $$\frac{\partial N}{\partial t} + \nabla \cdot (N\mathbf{V}) = -k_i n N$$

Discrete (predator-prey) model:

$$\frac{\partial nL}{\partial t} + nv = k_i nNL \qquad \int_0^L dx \frac{\partial}{\partial x}(NV) = -NV$$

$$\frac{\partial NL}{\partial t} - NV = -k_i nNL \qquad \int_0^L dx \frac{\partial}{\partial x}(nv) = nv$$

$$\omega^2 = k_i^2 n_0 N_0 = \frac{v_0 V_0}{L^2}$$

We were unable to get any oscillations in the continuous model: just one cycle of switching into stationary profiles. Underlying instability is important?

Axial mode instability with ionization

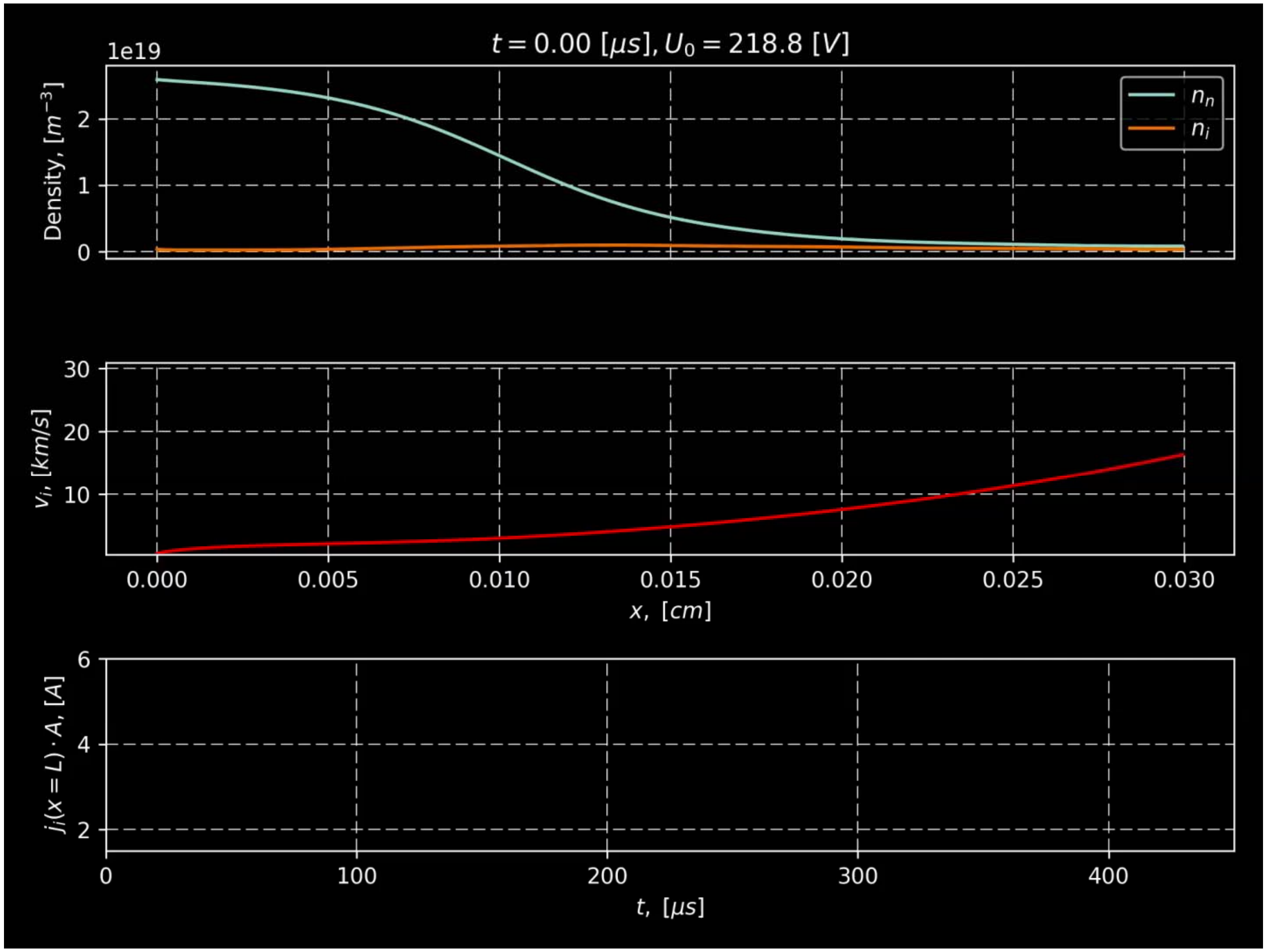
- Fixed velocity of neutrals
- Ballistic ions with momentum input from ioniz.
- Electron current is resistive
- Quasineutrality
- Total voltage is constrained

$$
\begin{aligned}
\frac{\partial N}{\partial t} + \frac{\partial}{\partial z}(NV) &= -k_i n N \\
\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n v_i) &= k_i n N \\
m_i\left(\frac{\partial}{\partial t} v_i + v_i \frac{\partial}{\partial z} v_i\right) &= eE + k_i N m_i (V - v_i) \\
J_T &\equiv J_i + J_e = e n v_i + \sigma_e E \\
\frac{\partial}{\partial z}(J_i + J_e) &= 0 \\
\int_0^L E \mathrm{d}z &= U_0
\end{aligned}
$$

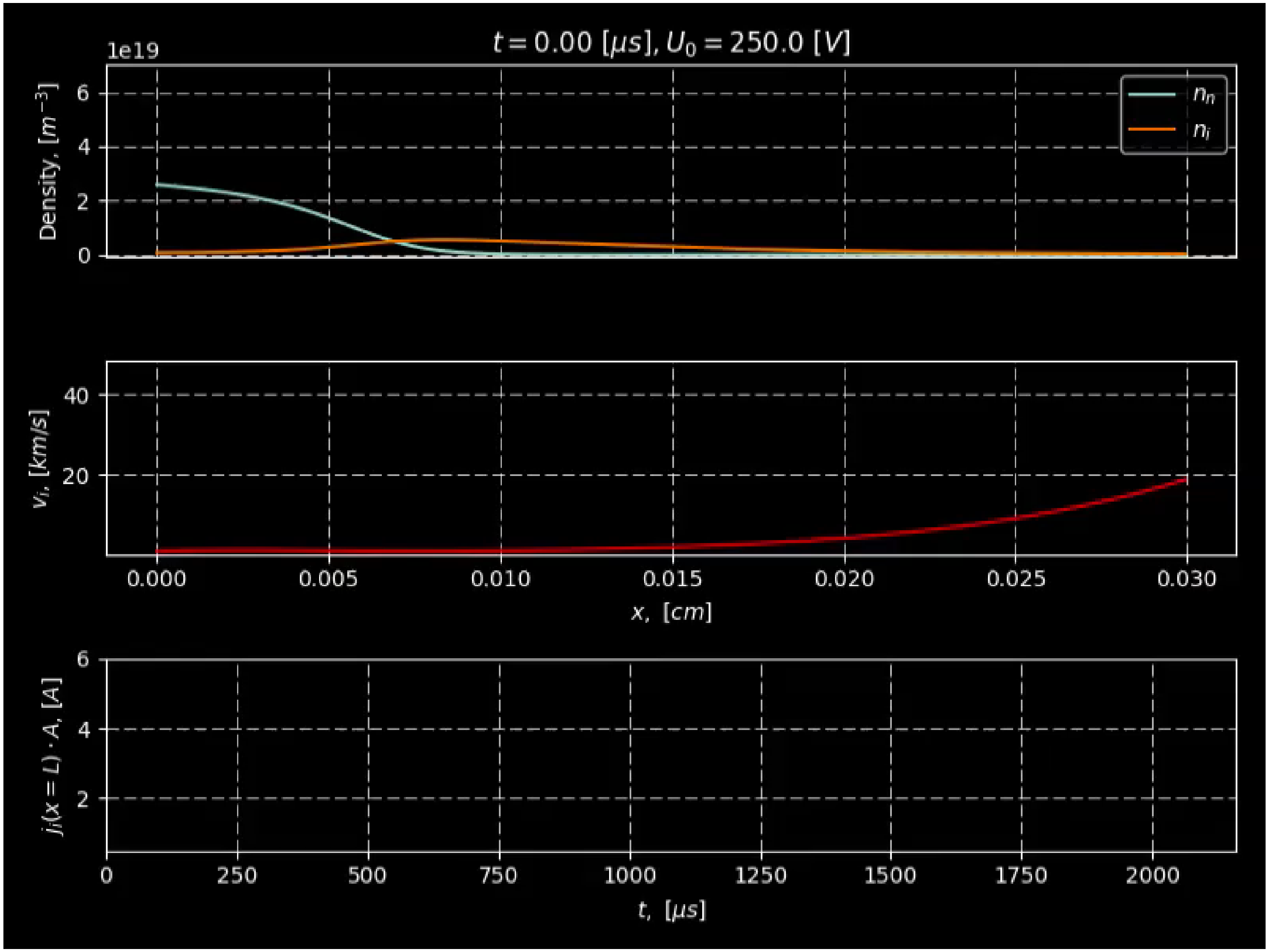
Morozov, Saveleev 1995
Boeuf, Garrigues, 1998
(but no electron energy equation)

Resistive (axial mode) instability is important for low frequency oscillations! Chable, Rogier 2005

Ionization and axial flow instability modes

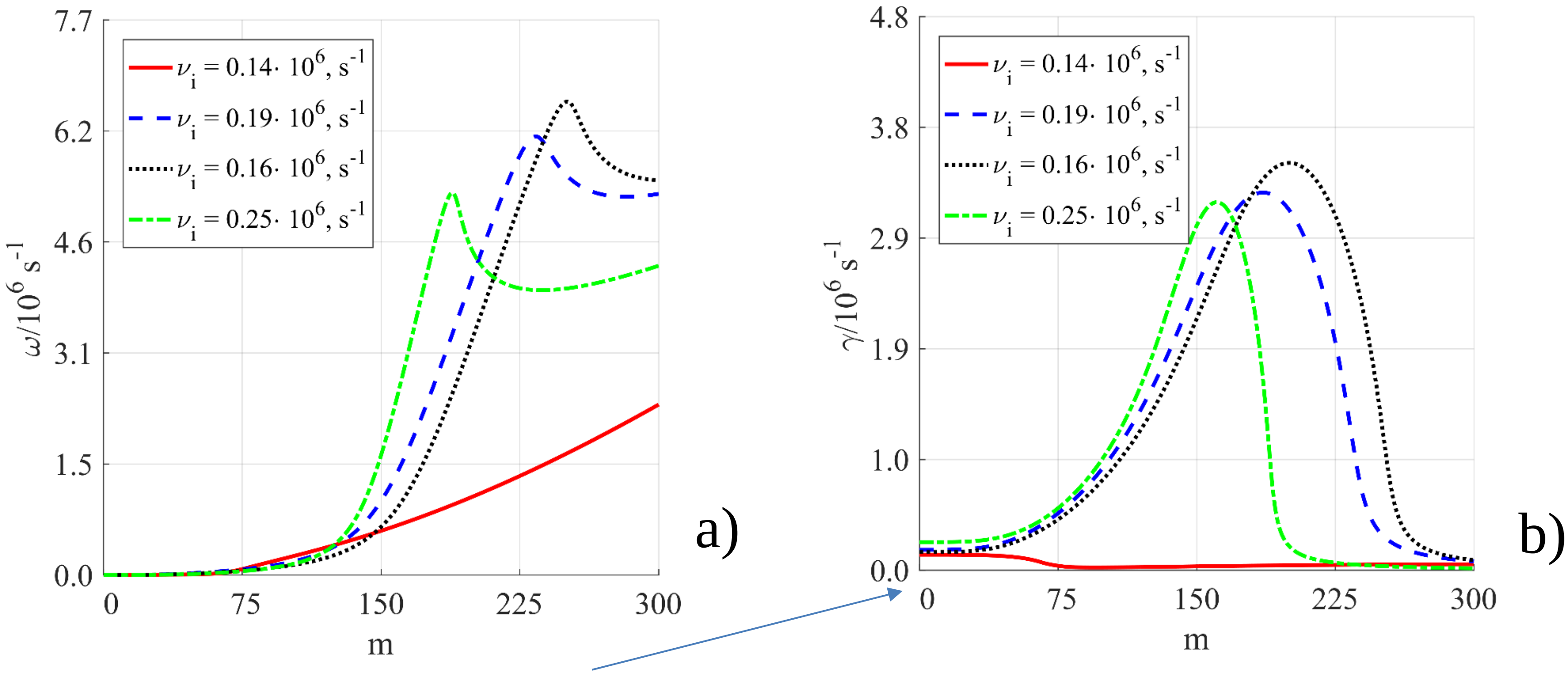


Ionization and axial flow instability modes



Effects of ionization on Simon-Hoh modes

$$\frac{k_y^2 c_s^2}{[\omega + i(\nu_i+\nu_x)]\left(\omega - i\nu_i - \frac{\nu_i\nu_d}{\omega+i\nu_d}\right)} = \frac{\omega_* - \omega_D + k_\perp^2\rho_e^2[\omega-\omega_0+i(\nu_i+\nu_{en})]}{\omega - i\nu_i - \frac{\nu_i\nu_d}{\omega+i\nu_d} - \omega_D + k_\perp^2\rho_e^2[\omega-\omega_0+i(\nu_i+\nu_{en})}$$



Scale-less instability at k=0 due to ionization; ionization zone propagation in the spoke? D. Escobar and E. Ahedo, PoP 2014, 2015

3D effects: Parallel electron dynamics along the magnetic field

$$m_e n \frac{dV_{\|e}}{dt} = -enE_{\|} - \nabla_{\|} p - \mathbf{b} \cdot \nabla \cdot \pi - m_e n \nu_{en} V_{\|e}$$

In general case, kinetic effects are important (included in the viscosity tensor). Strictly speaking parallel motion can not be described within the fluid model. Possible approaches are:

a) linear kinetic closures, Hammer-Perkins type, for collisionless wave-particle effects (Landau damping), future work!

b) Reduced, fluid like description are possible in two limits

- fluid limit $d/dt >> k_{\|} v_{Te}$
- Adiabatic limit $d/dt << k_{\|} v_{Te}$

Fluid limit $d/dt >> k_{\|} v_{Te}$

$$\frac{\tilde{n}}{n_0} = \frac{\omega_* - \omega_D + k_\perp^2 \rho_e^2 (\omega - \omega_0 + i\nu_{en})}{\omega - \omega_D - \omega_0 + k_\perp^2 \rho_e^2 (\omega - \omega_0 + i\nu_{en})} \frac{e\phi}{T_e}$$
$$- \frac{k_{\|}^2 v_{Te}^2}{(\omega - \omega_D - \omega_0)(\omega - 2\omega_D - \omega_0)} \frac{e\phi}{T_e}.$$

The last term in this expression results in the modified two-stream instability that occurs for small values of the parallel wave vector $\left(k_{\|}/k_\perp\right) \approx \left(m_e/m_i\right)^{1/2}$

Need to be included!

Adiabatic limit: $d/dt \ll k_{\|} v_{Te}$

$$-enE_{\|} - \nabla_{\|} p - m_e \nu_e V_{\| e} = 0$$

$$\frac{\tilde{n}}{n_0} = \frac{\omega_* - \omega_D + k_\perp^2 \rho_e^2 (\omega - \omega_0 + i\nu_{en}) + ik_{\|}^2 D_{\|}}{\omega - \omega_D - \omega_0 + k_\perp^2 \rho_e^2 (\omega - \omega_0 + i\nu_{en}) + ik_{\|}^2 D_{\|}} \frac{e\phi}{T_e}$$

What is a value for $k_{\|}$, $k_{\|} \approx \pi / L$?

However, typically $D_{\|} = {v_{Te}}^2 / \nu_e$ is pretty large:

${k_{\|}}^2 D_{\|} \gg \omega_0 ,...$ resulting in : $\dfrac{n_e}{n_0} = \dfrac{e\phi}{T_e}$

instability is suppressed

Rigorous approach: eigen-value problem along z (B)

Differential equation in z

$$\tilde{\Phi}^{(IV)} + Q\tilde{\Phi}^{II} - i(\omega - \omega_0) k_\perp^2 \frac{\nu}{v_{Te}^2} \tilde{\Phi} = 0$$

$$Q = \left(i(\omega - \omega_0)\frac{\nu}{v_{Te}^2} - k_\perp^2 - \frac{\omega_p^2}{v_{Te}^2}\left[1 - \frac{\Omega_p^2}{\omega^2}\right]^{-1} \right)$$

General solution $\Phi = \sum_{i=4} C_i \exp(k_i z)$ and boundary conditions

$$\Phi'(z = -L) = \Phi'(z = L) = n(z = -L) = n(z = L) = 0$$

Dispersion relation

$$\begin{matrix}
\lambda_1 \sinh(-\lambda_1 L) & \lambda_1 \cosh(\lambda_1 L) & \lambda_2 \sinh(-\lambda_2 L) & \lambda_2 \cosh(\lambda_2 L) \\
\lambda_1 \sinh(\lambda_1 L) & \lambda_1 \cosh(\lambda_1 L) & \lambda_2 \sinh(\lambda_2 L) & \lambda_2 \sinh(\lambda_2 L) \\
(\lambda_D^2 - \lambda_1^2)\cosh(\lambda_1 L) & (\lambda_D^2 - \lambda_1^2)\sinh(-\lambda_1 L) & (\lambda_D^2 - \lambda_2^2)\cosh(\lambda_2 L) & (\lambda_D^2 - \lambda_2^2)\sinh(-\lambda_2 L) \\
(\lambda_D^2 - \lambda_1^2)\cosh(\lambda_1 L) & (\lambda_D^2 - \lambda_1^2)\sinh(\lambda_1 L) & (\lambda_D^2 - \lambda_2^2)\cosh(\lambda_2 L) & (\lambda_D^2 - \lambda_2^2)\sinh(-\lambda_2 L)
\end{matrix}$$

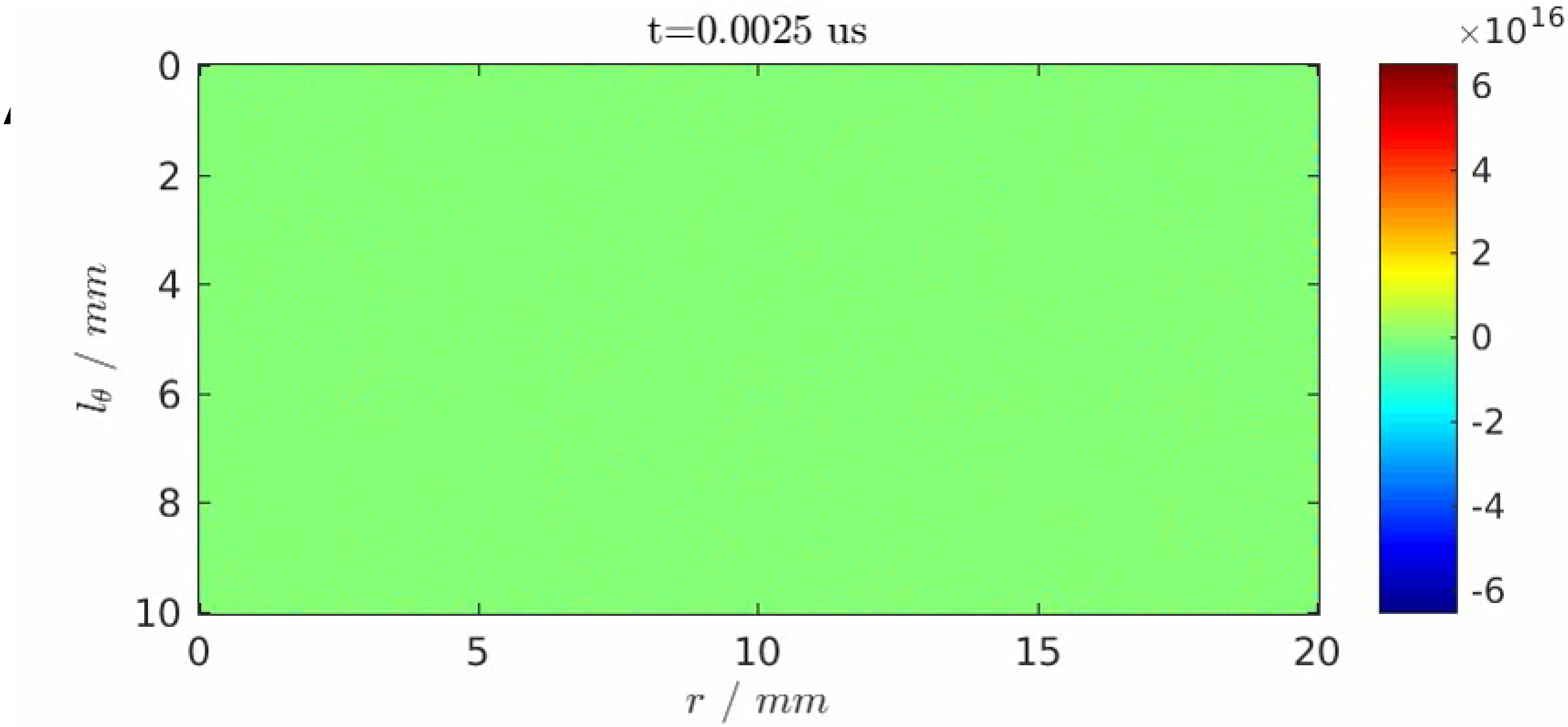
Solution

$$\begin{aligned}
\tilde{\Phi} &= -0.269447 - 0.01103z + Re\left[-(0.317965 - 0.772319i)cosh(kz)\right] \\
&\quad + Re\left[(4.25007 \times 10^{-16} - 2.42428 \times 10^{-16})sinh(kz)\right]
\end{aligned}$$

Double degenerate root k=0

Compared to the local theory with a naive estimate $k_{\|} \approx \pi / L$, the imaginary part of ω is approximately an order magnitude larger, equivalent to the much small value of $k_{\|} << \pi / L$

Sheath boundary conditions also give similar result: The inner part of plasma is insulated from boundaries and have much smaller effective k



Cross-field current driven ion acoustic instability, Barrett, PRL 1972

Plasma perturbations look much longer than L

for large values of b. Attempts either to measure or to prescribe k_z for small b show that the instability runs at k_z values which are smaller than π/L. Presumably, the wave electric fields fall rapidly to zero at the plasma boundary sheaths, but have wavelengths greater than $2L$ along $\vec{B}$ in the bulk of the plasma. Since accurate measure-

Summary

- Coupled Simon-Hoh, lower- hybrid and ion sound instabilities were studied analytically and in advanced fluid model simulations (BOUT++)
- Nonlinear saturation of instabilities driven by ExB and ion flows, collisions and density gradients has been obtained .
- Strongly intermittent anomalous current was found well above classical values, consistent with PIC simulations and experiment (Penning discharge)
- Large scale structures in the vorticity have been found correlated with enhanced anomalous current
- Slow growing large amplitude structures were found coexisting with small scale fluctuations. Linear and nonlinear theory of the current flow modes
- Nonlocal eigen-mode problem along the magnetic field indicates much lower values of the effective wave-vector, also for sheath boundary conditions

Lower-hybrid waves

Plasma density and electric field fluctuations: cold plasma

Ion inertia

$$m_i \frac{\partial \tilde{\mathbf{V}}_i}{\partial t} = e\mathbf{E} = -e\nabla\phi$$

Restoring force is provided by the compressibility of the electron motion across the magnetic (electron inertia)

$$m_e \frac{\partial \tilde{\mathbf{V}}_e}{\partial t} = -e\left(\mathbf{E} + \mathbf{V_e} \times \mathbf{B}\right)$$

$$\tilde{\mathbf{V}}_\mathbf{e} = \frac{\tilde{\mathbf{E}} \times \mathbf{B}}{B^2} - \frac{1}{\omega_{ce} B}\frac{\partial}{\partial t}\tilde{\mathbf{E}}_\perp$$

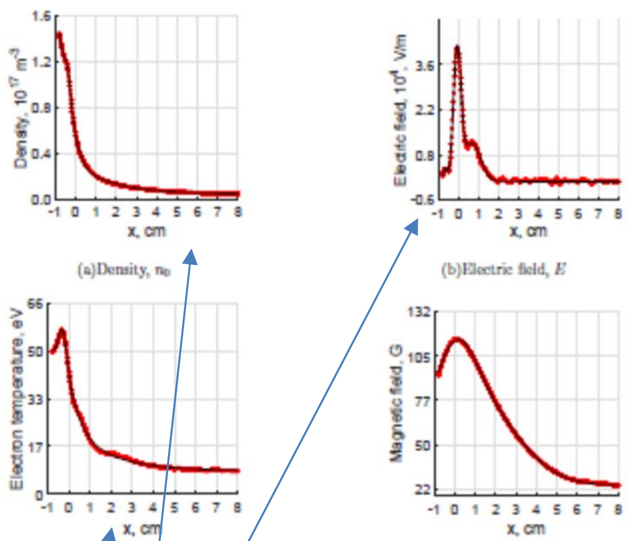
$$\nabla \cdot \tilde{\mathbf{V}}_\mathbf{e} = -\frac{1}{\omega_{ce} B}\frac{\partial}{\partial t}\nabla \cdot \tilde{\mathbf{E}}_\perp \neq 0$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot \left(n\tilde{\mathbf{V}}_i\right) = 0 \quad \longleftrightarrow \quad \frac{\partial \tilde{n}}{\partial t} + \nabla \cdot \left(n\tilde{\mathbf{V}}_e\right) = 0$$

Balance of ion inertia against the electron transverse inertia: lower-hybrid wave →

$$\omega^2 = \omega_{ce}\omega_{ci}$$

Nonlinear, multi-scale, self-organization problem



Magnetic field is prescribed externally

But density, temperature and electric field are self-consistent

Density/temperature gradients, ExB drift due to electric field (free energy for instabilities)

ionization

Global balance/constraints, e.g. voltage drop/power/flow rate

Electron current

Heating

Anomalous transport at global/large scales:

Fluctuations/turbulence
fast/small scales

Hall plasmas with $\mathbf{E}\times\mathbf{B}$ drift

- Observed fluctuations frequencies: 1 kHz-1 MHZ

  $B = 50 \div 100$ G $\quad T_e = 10 \div 30$ eV $\quad T_i < 1$ eV $\quad E_0 = 10^4$ V/m

- Magnetized electrons $\quad \omega < \omega_{ce} \quad \rho_e < L$
- Unmagnetized ions $\quad \omega > \omega_{ci} \quad \rho_i > L$
- Electrons are magnetically controlled; motion across the magnetic field is constrained due to small Larmor radius; stationary drift $\quad \mathbf{V_E} = \mathbf{E}\times\mathbf{B}/B^2$
- Ions are weakly affected by the magnetic field; controlled by the electric field; can be accelerated/extracted etc
- Density, temperature and magnetic field are inhomogeneous -> coupling to drift wave physics